Seat No.:________

GANPAT UNIVERSITY
B.Tech. Sem.-I (ALL) Exam; Dec.- 2010
HS-101, Engg. Mathematics - I

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate full marks of question.

SECTION – I

Question-1

A Verify Roll's theorem for the function $f(x)=(x-3)\cos x$; $\left[\frac{\pi}{2}, 3\right]$ .Also prove that one root of the equation : $\cot x = x-3$ lies in $\left(\frac{\pi}{2}, 3\right)$ (03)

B **Attempt any two** (08)

(a) Verify Cauchy's mean value theorem and find C for the functions $f(x)=\sin x$, $g(x)=\cos x$; $[0, \pi/2]$

(b) If $v=\left(x^2+y^2+z^2\right)^{-1/2}$, then prove that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$

(c) If $z=f(u,v)$ *where* $u=lx+my$, $v=ly-mx$, then prove that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\left(l^2+m^2\right)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right)$

Question-2 **Attempt any three** (12)

(a) If $y=\frac{3x+1}{(x+1)^2(x-2)}$, find y_n.

(b) If $y=e^{\tan^{-1}x}$, then prove that $\left(1+x^2\right)y_{n+2}+\left[2(n+1)x-1\right]y_{n+1}+n(n+1)y_n=0$

(c) Expand $\sin x$ in powers of $\left(x-\frac{\pi}{2}\right)$ up to fourth power. hence evaluate $\sin 91^\circ$ up to four decimal places

(d) Evaluate : (1) $\lim\limits_{x\to 0}\frac{\tan x - x}{x^2\tan x}$ (2) $\lim\limits_{x\to 1}(2-x)^{\tan(\pi x/2)}$

Question-3 **Attempt any three** (12)

(a) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 4u-\sin 2u$

(b) If $u=xyz$, $v=xy+yz+zx$, $w=x+y+z$ then P.T. $\frac{\partial(u,v,w)}{\partial(x,y,z)}=(x-y)(y-z)(z-x)$

(c) Find the extreme values of x^3+y^3-3axy

(d) The Horse power required to propel a steamer varies as the cube of the velocity and the square of length. If there is 3% increase in velocity and 4% increase in length then find the percentage increase in Horse power.

SECTION – II

Question-4

A Show that the points $(-4,9,6)$, $(-1,6,6)$, $(0,7,10)$ form a right-angled triangle. **(03)**

B <u>**Attempt any two**</u> **(08)**

(a) Find the rank of a matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

(b) Find the inverse of a matrix by Gauss-Jordan method $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

(c) Test for the consistency and solve the system

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=5$$

Question-5 <u>**Attempt any three**</u> **(12)**

(a) Test the convergence of $\frac{2^p}{1^q}+\frac{3^p}{2^q}+\frac{4^p}{3^q}+\frac{5^p}{4^q}+------$

(b) Test the convergence of $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\frac{x^4}{4.5}+---$

(c) Test the convergence of $\sum_{n=1}^{\infty}\frac{\left[(n+1)x\right]^n}{n^{n+1}}$

(d) Test the convergence of $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^2}$

Question-6 <u>**Attempt any three**</u> **(12)**

(a) Find eigen-values and eigen -vectors of $\begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

(b) Check wheather the given vectors are L.D. or L.I. ? If L.D. then find a relation between them

$x_1=(1,2,4)$, $x_2=(2,-1,3)$, $x_3=(0,1,2)$, $x_4=(-3,7,2)$

(c) Investigate for what values of λ & μ the equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

have (i) no solution (ii) a unique solution (iii) an infinite no of solutions

(d) Find the equation of a plane which passes through the point (3 , -3 , 1) and is parallel to the plane 2x+3y+5z+6=0