K.P.A

Seat No.:_______

GANPAT UNIVERSITY

B.Tech Sem-I (ALL) Examination

Nov./Dec - 2011

Subject : HS101;Engineering Mathematics – I

[Time:03 hours] [Marks :70]

Instructions:

(1)All questions are compulsory.

(2)Answers to the two sections should be written in separate answer books.

(3)Figures to the right indicate full marks of the questions.

SECTION –I

Question-1 Attempt the following. (12)

(a) Using Maclaurin's expansion.theorem prove that $\sec x = 1 + \frac{x^2}{2!} + 5\frac{x^4}{4!} + ---$

(b) If $y = \log\left(x + \sqrt{x^2 + 1}\right)$, prove that $\left(1 + x^2\right) y_{n+2} + \left(2n+1\right) x y_{n+1} + n^2 y_n = 0$.

(c) Evaluate : (1) $\lim_{x\to 1}\left(\frac{1 + \cos \pi x}{(x-1)^2}\right)$ (2) $\lim_{x\to \pi/2}\left(1 - \sin x\right)\tan x$

Question-1 OR (12)

(a) Expand the function $\tan^{-1} x$ by Taylor's theorem in powers of $(x-1)$ up to $(x-1)^4$ terms.

(b) If $x = \sin\theta$, $y = \cos m\theta$ show that $\left(1 - x^2\right) y_{n+2} - \left(2n+1\right) x y_{n+1} = (n^2 - m^2) y_n$

(c) Evaluate : (1) $\lim_{x\to 0}\frac{\log x}{\cot x}$ (2) $\lim_{x\to 1}(x-1)^{(x-1)}$

Question-2 Attempt the following.

(a) If $u = x^2 y + y^2 z + z^2 x$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2$ (4)

(b) If $x = u(1-v)$, $y = uv$, evaluate $J = \frac{\partial(x, y)}{\partial(u, v)}$ and $J' = \frac{\partial(u, v)}{\partial(x, y)}$ and verify $JJ' = 1$ (4)

(c) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$ prove that $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$ (3)

Question-2 OR

(a) If $v = (1 - 2xy + y^2)^{-1/2}$, prove that $\frac{\partial}{\partial x}\left[(1 - x^2)\frac{\partial v}{\partial x}\right] + \frac{\partial}{\partial y}\left[y^2 \frac{\partial v}{\partial y}\right] = 0$ (4)

(b) If $u = xyz$, $v = xy + yz + zx$, $w = x + y + z$, prove that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = (x - y)(y - z)(z - x)$ (4)

(c) Verify Roll's theorem for the function $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$ (3)

Question-3 Attempt the following. (12)

(a) If $u = \sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{400}\left[\tan^2 u - 19\right]$

(b) Examine the extreme values of the function $f(x,y) = x^2 - 2xy - \frac{1}{3}y^3 - 3y$.

(c) Using partial fraction find n^{th} derivative y_n for $y = \dfrac{2x-3}{x(x^2-1)}$.

Question-3 **OR** **(12)**

(a) If $u = \dfrac{x^2 y^2}{x+y}$, prove that (a) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3u$ (b) $x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial x \partial y} = 2\dfrac{\partial u}{\partial x}$

(b) Show that of all rectangular parallelopiped with given volume, the cube has the least surface area

(c) If $y = \dfrac{x}{(x^2+a^2)}$, prove that $y_n = \dfrac{(-1)^n n! \cos(n+1)\theta . \sin^{n+1}\theta}{a^{(n+1)}}$, where $\theta = \tan^{-1}\left(\dfrac{a}{x}\right)$

SECTION – II

Question-4 **Attempt the following.** **(12)**

(a) Find the inverse of a matrix by Gauss-Jordan method $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

(b) Test for the consistency and solve the system $\begin{matrix} x-3y-8z=-10 \\ 2x+5y+6z=13 \\ 3x+y-4z=0 \end{matrix}$ if it is consistent.

(c) Using normal form, find the rank of a matrix $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

Question-4 **OR** **(12)**

(a) Find the inverse of a matrix by Gauss-Jordan method $\begin{bmatrix} 4 & 3 & 3 \\ -1 & 0 & -1 \\ -4 & -4 & -3 \end{bmatrix}$

(b) Investigate for what values of λ & μ the equations $\begin{matrix} 2x+3y+5z=9 \\ 7x+3y-2z=8 \\ 2x+3y-\lambda z=\mu \end{matrix}$ have (i) no solution (ii) a unique solution (iii) an infinite no of solutions

(c) Show that there exist at least one plane passing through the points $(1,2,3)$, $(3,-2,1)$ and $(1,-6,-5)$

Question-5 **Attempt the following.**

(a) Test the convergence of $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + ---$ **(04)**

(b) Test the convergence of $\sum_{n=1}^{\infty} \dfrac{[(n+1)x]^n}{n^{n+1}}$ **(04)**

(c) Test the convergence of $\frac{1}{3} + \frac{2!}{3^2} + \frac{3!}{3^3} + ---$ **(03)**

OR

(a) Test the convergence of (i) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ (ii) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^3-n-1}$ **(04)**

(b) Test the convergence of $\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+---$ **(04)**

(c) Test the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n-1)}{2.4.6............2n}$ **(03)**

Question-6 Attempt any three **(12)**

(a) Find eigen values and eigen vectors of matrix $\begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

(b) Are the given vectors: $x_1=[1,2,4], x_2=[2,-1,3], x_3=[0,1,2], x_4=[-3,7,2]$ Linearly dependent? If so express x_1 as Linear Combination of the others.

(c) Using triangularization process find the rank of a matrix $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 2 & 3 & 5 & 1 \\ 1 & 2 & 3 & 2 \end{bmatrix}$

(d) Trace the curve $r = 2(1+\cos\theta)$

End of Paper