G.B

Seat No. _____

GANPAT UNIVERSITY

B.Tech. Sem. – I (ALL) CBCS Regular Exam. Dec – 2012

Sub : (HS – 101) Engineering Mathematics – I

Time : 3 hrs **Total marks : 70**

Instruction : (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que 1 **(12)**

(A) Verify Roll's theorem for the function $f(x) = (x-3)\cos x : \left[\frac{\pi}{2}, 3\right]$

Also prove that one root of the equation : $\cot x = x - 3$ lies in $\left(\frac{\pi}{2}, 3\right)$

(B) If $v = r^m$ where $r^2 = x^2 + y^2 + z^2$ then P.T.

$$v_{xx} + v_{yy} + v_{zz} = m(m+1)\, r^{m-2}$$

(C) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ then P.T. $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$

OR

Que 1 **(12)**

(A) Using LMV theorem P.T. for $a < b$; $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$

Hence deduce that : $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$

(B) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then P.T. $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

(C) If $u = f(x, y)$ where $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ then P.T.

$$\left[\frac{\partial u}{\partial r}\right]^2 + \frac{1}{r^2}\left[\frac{\partial u}{\partial \theta}\right]^2 = \left[\frac{\partial f}{\partial x}\right]^2 + \left[\frac{\partial f}{\partial y}\right]^2$$

Que 2

(A) If $u = \sin^{-1}\left[\frac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt{x} + \sqrt{y}}\right]^{1/2}$ then P.T. **(03)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}\left[13 + \tan^2 u\right]$$

(B) Find the extreme value of : $x^2 - 2xy + \frac{y^3}{3} - 3y$ **(04)**

(C) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$. **(04)**

OR

Que 2

(A) A rectangular box open at the top is to have a volume of 32 cubic ft. Find it's dimensions; if the total surface is a minimum. **(03)**

(B) Find the possible Percentage error in measuring the parallel resistance r of two resistance r_1 and r_2 ; If r_1 & r_2 are both in error by 2 %. **(04)**

(C) If $\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$ then P.T. $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$ **(04)**

Que 3 <u>**Attempt any three**</u> **(12)**

(A) If $y = \dfrac{1}{x^2 + a^2}$ then P.T. $y_n = \dfrac{(-1)^n\, n!}{a^{n+2}} \sin^{n+1}\theta\, \sin(n+1)\theta$

(B) If $y = e^{a\sin^{-1}x}$ then P.T. $(1 - x^2)\, y_{n+2} - (2n+1)\, x\, y_{n+1} - (n^2 + a^2)\, y_n = 0$

(C) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ up to 4^{th} power. Also find the value of $\sin 91°$ up to four decimal places.

(D) Evaluate : (1) $\lim\limits_{x\to 1} \dfrac{x^x - x}{x - 1 - \log x}$ (2) $\lim\limits_{x\to 0} \left[\dfrac{a^x + b^x + c^x + d^x}{4}\right]^{1/x}$

<u>Section – II</u>

Que 4 **(12)**

(A) Find the rank of a matrix : $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

(B) Find Eigen – values and Eigen – vectors of : $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(C) Test for the consistency and solve the system : $\begin{cases} x - 3y - 8z = -10 \\ 2x + 5y + 6z = 13 \\ 3x + y - 4z = 0 \end{cases}$

OR

Que 4 **(12)**

(A) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ then find the rank using Normal form of Matrix.

(B) Find Eigen – values and Eigen – vectors of : $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(C) Test for the consistency and solve the system : $\begin{cases} 2x - y + z = 9 \\ 3x - y + z = 6 \\ 4x - y + 2z = 7 \\ -x + y - z = 4 \end{cases}$

Que 5

(A) Show that the points (0,4,1) , (2,3,−1) , (4,5,0) , (2,6,2) are the vertices of a square. **(03)**

(B) Find the inverse of a matrix by Gauss – Jordan method $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ **(04)**

(C) Investigate for what values of λ & μ the equations $\begin{cases} 2x + 3y + 5z = 9 \\ 7x + 3y - 2z = 8 \\ 2x + 3y - \lambda z = \mu \end{cases}$ **(04)**

have (i) no solution (ii) a unique solution and (iii) an infinite no of solutions

OR

Que 5

(A) Find the equation of a plane which passes through the point $(1, -3, 1)$ and is parallel to the plane : $2x + y - z + 10 = 0$ **(03)**

(B) Find the inverse of a matrix by Gauss – Jordan method $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ **(04)**

(C) Check whether the given vectors are L. D. or L. I.? If L. D. then find a relation between them : $x_1 = (1,0,2,1)$, $x_2 = (3,1,2,1)$, $x_3 = (4,6,2,-4)$, $x_4 = (-6,0,-3,-4)$ **(04)**

Que 6 **<u>Attempt any three</u>** **(12)**

(A) Test the convergence of $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} + \cdots\cdots\cdots$

(B) Test the convergence of $\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \frac{x^4}{4\cdot 5} + \cdots\cdots$

(C) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$

(D) Test the convergence of $\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

END OF PAPER