Seat No. ______

# GANPAT UNIVERSITY

**B. Tech. Sem. – I (ALL) CBCS Remedial Exam. Dec – 2013**

**Sub : (2HS101) Engineering Mathematics – I**

**Time : 3 hrs** **Total marks : 70**

**Instruction :** (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

## Section – I

**Que 1** **(12)**

(A) Verify Roll's theorem for the function $f(x) = (x-4)\log x \; : x \in R$.
Also prove that one root of the equation : $x\log x + x = 4$ lies in $(1, 4)$.

(B) If $v = (x^2+y^2+z^2)^{-1/2}$ then P.T. $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$

(C) If $u = f\left(\frac{x}{z}, \frac{y}{z}\right)$ then P.T. $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

**OR**

**Que 1** **(12)**

(A) Verify Cauchy's mean value theorem for the function : $\begin{cases} f(x) = \sin x \\ g(x) = \cos x \end{cases}$ ; $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Also prove that for $[a, b] \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ; there exists $c \in (a, b)$ s.t. $c = \frac{a+b}{2}$

(B) If $u = \tan^{1}\left(\frac{xy}{\sqrt{1+x^2+y^2}}\right)$ then Prove that $\frac{\partial^2 u}{\partial x\,\partial y} = [1+x^2+y^2]^{-3/2}$

(C) If $u = f(r)$ ; where $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ then P.T.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

**Que 2**

(A) State Euler's theorem on homogeneous function. If $u = \tan^{-1}[x^2 + 2y^2]$ **(03)**

then P.T. $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u\cos 3u$

(B) Find the extreme value of : $\sin x \sin y \sin(x+y)$ ; $0 < x, y < \pi$ **(04)**

(C) Using Langrage's undetermined multiplier's method ; Find the maximum volume **(04)**
which can be inscribed in the ellipsoid : $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

**OR**

**Que 2**

(A) A rectangular box open at the top is to have a volume of 32 cubic ft. Find it's dimensions ; if the total surface is a minimum. **(03)**

(B) The Horse power required to propel a steamer varies as the cube of the velocity and the square of length. If there is 3% increase in velocity and 4% increase in length then find the percentage increase in Horse power. (04)

(C) If $\begin{cases} u = xyz \\ v = xy + yz + zx \\ z = x + y + z \end{cases}$ then Prove that $\dfrac{\partial(u, v, w)}{\partial(x, y, z)} = (x - y)(y - z)(z - x)$. (04)

**Que 3** <u>**Attempt any three**</u> (12)

(A) If $y = \dfrac{3x - 2}{(x+1)^2\,(x+3)}$ then find $y_n$

(B) If $y = \left(x + \sqrt{1 + x^2}\right)^m$ then Prove that

$$(1 + x^2)y_{n+2} + (2n+1)\,x\,y_{n+1} + (n^2 - m^2)y_n = 0$$

(C) Using Maclaurin's theorem Prove that : $\log(1 + e^x) = \log 2 + \dfrac{x}{2} + \dfrac{x^2}{8} - \dfrac{x^4}{192} + \cdots$

(D) Evaluate : (1) $\displaystyle\lim_{x \to y} \frac{x^y - y^x}{x^x - y^y}$ (2) $\displaystyle\lim_{x \to 0} \left[\sec^2 x\right]^{\cot^2 x}$

## Section - II

**Que 4** (12)

(A) Find the rank of Matrix $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$

(B) Find Eigen value and Eigen vectors of Matrix $\begin{bmatrix} 0 & 0 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

(C) Test the consistency and solve the equation:

$x + y + 2z = 9, \quad 2x + 4y - 3z = 1, \quad 3x + 6y - 5z = 0$

OR

**Que 4** (12)

(A) Using the normal form find rank of matrix $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

(B) Find Eigen value and Eigen vectors of Matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

(C) Test the consistency and solve the equation:

$x + y + z = 6, \quad x + 2y + 3z = 14, \quad 2x + 4y + 7z = 30$

**Que 5**

(A) If $A = \dfrac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, show that $A.A' = A'.A = I$ (03)

(B) By using Gauss-Jordan method, find the inverse of the given matrix (04)

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

(C) For which values of $\lambda$ and $\mu$ the following system have (i) no solution (ii) unique solution and (iii) an infinite no. of solutions (04)
$2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y - \lambda z = \mu$

**OR**

**Que 5**

(A) If $D = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ find the value of $D^2 + 2D + I$ (03)

(B) Find the inverse of the given matrix using Gauss-Jordan $\begin{bmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{bmatrix}$ (04)

(C) Check whether the following vectors are L.I or L.D. In case of L.D, express one of the vector as a linear combination of others. $x_1 = (1, -1, 1)$, $x_2 = (2, 1, 1)$, $x_3 = (3, 0, 2)$ (04)

**Que 6** **Attempt any three** (12)

(A) Test the convergence for following series $\frac{2}{1.2.3} + \frac{4}{4.5.6} + \frac{6}{7.8.9} + - - -$

(B) Test the convergence for following series $\frac{1}{\sqrt{1}}x + \frac{1}{\sqrt{2}}x^2 + \frac{1}{\sqrt{3}}x^3 + - - - + \frac{1}{\sqrt{n}}x^n + - - -$

(C) Test the convergence for following series $\sum_{n=1}^{\infty}\left(1 + \frac{1}{n}\right)^{-n^2}$

(D) Test the convergence for following series $\sum_{n=2}^{\infty}\frac{1}{n \log n}$

**END OF PAPER**