Seat No.______

GANPAT UNIVERSITY

B.Tech. Sem.-I (ALL) CBCS REGULAR EXAMINATION, DECEMBER - 2014

SUBJECT: 2HS101 - CALCULUS

Time: 3 hrs **Total marks: 60**

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que-1 Answer the following

(a) Find the n^{th} derivatives of $y = \frac{x}{x^2+a^2}$ 4

(b) Using Maclaurin's theorem Prove that : $\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \dots\dots\dots$ 3

(c) Evaluate : (1) $\lim_{x\to 0}\frac{\tan x - \sin x}{\sin^3 x}$ (2) $\lim_{x\to 0}(\cos x)^{\cot 2x}$ 3

OR

Que-1 Answer the following.

(a) If $y = (\sin^{-1} x)^2$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ 4

(b) Expand $\tan(x + \frac{\pi}{4})$ up to x^4 and hence evaluate $\tan 50^\circ$. 3

(c) Evaluate: (1) $\lim_{x\to 0}\frac{\log(\tan 2x)}{\log(\tan x)}$ (2) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$ 3

Que-2 Answer the following.

(a) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u \cos 3u$ 4

(b) If $u = \log\left(\frac{x^3+y^3}{xy}\right)$ then prove that $u_{xy} = u_{yx}$ 3

(c) Find extreme values of $x^2 - 2xy + \frac{1}{3}y^3 - 3y$ 3

OR

Que-2 Answer the following.

(a) If $u = xyz,\ v = xy + yz + zx,\ w = x + y + z$ then prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = (x-y)(y-z)(z-x)$ 4

(b) If $y = f(x+ct) + g(x-ct)$ prove that $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ 3

(c) Verify Euler's theorem for $u = ax^2 + 2hxy + by^2$ 3

Que-3 (a) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ then P.T. $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ 4

(b) **Attempt any two** 6

(i) If $v = \frac{1}{r}$, where $r^2 = x^2 + y^2 + z^2$ show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$

(ii) If $u = f(x-y, y-z, z-x)$ then Prove that : $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

(iii) If $x = u(1-v)$ and $y = u \cdot v$ then evaluate $J = \frac{\partial(x,y)}{\partial(u,v)}$

Section – II

Que-4 Answer the following.

(a) Evaluate $\int_0^1 x^5 \sin^{-1} x\, dx$ using Reduction formula. **4**

(b) Define Error function and prove that $erf(\infty) = 1$ **3**

(c) Evaluate in terms of Elliptic integral : $\int_0^{\pi/2} \frac{dx}{\sqrt{1+4\sin^2 x}}$ **3**

OR

Que-4 Answer the following.

(a) Define Beta & Gamma function and Evaluate $\int_0^2 x^4 \left(8-x^3\right)^{-\frac{1}{3}} dx$ **4**

(b) Prove that : (1) $erf(x) + erf_c(x) = 1$ (2) $erf(-x) = -erf(x)$ **3**

(c) Evaluate in terms of Elliptic integral : $\int_0^{\pi/2} \sqrt{\cos x}\, dx$ **3**

Que-5 Answer the following.

(a) Evaluate $\iint_R xy\, dxdy$, Where R is the region bounded by the circle $x^2 + y^2 = a^2$. **4**

(b) Evaluate $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}}\, dy\, dx$ **3**

(c) Evaluate $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x+y+z)\, dxdydz$ **3**

OR

Que-5 Answer the following.

(a) Change the order of integration and evaluate $\int_0^{\infty} \int_0^{x} x e^{-\frac{x^2}{y}}\, dxdy$ **4**

(b) Evaluate : $\int_0^{\frac{\pi}{2}} \int_0^{a} r^4 \sin^2\theta\, dr\, d\theta$ **3**

(c) Evaluate $\int_0^{a} \int_0^{x} \int_0^{(x+y)} e^{x+y+z}\, dzdydx$ **3**

Que-6 (a) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}\, dxdy$ by changing the Cartesian coordinates into polar coordinate. **4**

(b) Attempt any two. **6**

(i) Evaluate using Reduction formula $\int_5^7 \sqrt{(x-3)(7-x)}\, dx$.

(ii) Define Gamma function and prove that $\int_0^{\infty} e^{-ax} x^{n-1} dx = \frac{\Gamma n}{a^n}$

(iii) Find the area between the Parabolas $y^2 = 4ax$ & $x^2 = 4ay$.

END OF PAPER