Evening
Date: 14/12/2015

Ganpat University

B.Tech. Sem-I&II CBCS (New) Regular/Remedial (All Branches)Theory Examination, Nov - December 2015

Sub: 2HS101 CALCULUS

Total Marks: 60

Time: 3 Hours

Instructions: (1) This Question paper has two sections. Attempt each section in separate answer book.
(2) Figures on right indicate marks.
(3) Be precise and to the point in answering the descriptive questions.

SECTION-I

Question-1 Attempt the following.

(a) Using Maclaurin's expansion.theorem prove that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + - - -$ **03**

(b) If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **03**

(c) Evaluate : (1) $\lim_{x\to 0}\left(\frac{2x - x\cos x - \sin x}{2x^3}\right)$ (2) $\lim_{x\to \pi/2}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$ **04**

OR

Question-1

(a) If $y = e^{ax}\sin(bx+c)$, prove that $y_n = (a^2+b^2)^{n/2} e^{ax} \sin\left(bx + c + n\tan^{-1} b/a\right)$. **03**

(b) Find n^{th} derivative of the functions. (1) $\cos x \cos 2x \cos 3x$ (2) $\frac{2x-1}{x^2-5x+6}$ **03**

(c) If $y = x^{n-1}\log x$, prove that (1) $xy_1 = x^{n-1} + (n-1)y$ (2) $y_n = \frac{(n-1)!}{x}$ **04**

Question-2 Attempt the following.

(a) If $u = e^{xyz}$, prove that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$ **03**

(b) Examine the extreme values of the function $x^2 - 2xy - \frac{1}{3}y^3 - 3y$. **03**

(c) If $x = u(1-v)$, $y = uv$, evaluate $J = \frac{\partial(x,y)}{\partial(u,v)}$ and $J' = \frac{\partial(u,v)}{\partial(x,y)}$ and verify $JJ' = 1$ **04**

OR

Question-2

(a) If $v = (1 - 2xy + y^2)^{-1/2}$, prove that $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = y^2 v^3$ **03**

(b) If $z = f(x,y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$ **03**

(c) Use Lagrange method of undetermined multipliers to find the shortest distance from the point (1,2,2) to the sphere $x^2 + y^2 + z^2 = 16$ **04**

Question-3

(A) If $u = \tan^{-1}\left(x^2 + 2y^2\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u . \cos 3u$ **04**

(B) **Attempt any two.** **06**

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

(b) If $V = r^m$, where $r^2 = x^2 + y^2 + z^2$ prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = m(m+1).r^{m-2}$

(c) Evaluate : $\lim_{x\to 1}\left(1 - x^2\right)^{1/\log(x-1)}$:

SECTION-II

Question-4 **Attempt the following.**

(a) Define Gamma function. Prove that $\overline{)m} = 2\int_0^\infty e^{-x^2} x^{2m-1} dx$ 03

(b) Prove that $erf(x) + erf_c(x) = 1$ 03

(c) Define Beta function. Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx$ 04

Question-4 **OR**

(a) Evaluate: $\int_0^{\pi/2} \frac{d\theta}{\sqrt{9+4\sin^2\theta}} dx$ 03

(b) Evaluate in terms of Elliptic integral $\int_0^\infty \frac{dx}{\sqrt{(x^2+1)(x^2+4)}}$ 03

(c) Prove that (i) $erf(-x) = -erf(x)$ (ii) $erf(\infty) = 1$ 04

Question-5 **Attempt the following.**

(a) Evaluate : $\int_0^\pi (1+\cos x)^4 \, dx$ 03

(b) Using reduction formula find $\int_0^\infty \frac{x^2}{\sqrt{(1+x)^8}} dx$ 03

(c) Evaluate : $\int_{-1}^{1}\int_0^x\int_{x-y}^{x+y} (z-2x-y)\, dzdydx$ 04

Question-5 **OR**

(a) Using reduction formula find $\int_0^\pi x\sin^6 x.\cos^4 x \, dx$ 03

(b) If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$ prove that $I_n = \frac{1}{n-1} - I_{n-2}$ 03

(c) Evaluate : $\int_{-\pi/2}^{0}\int_0^{2\sin\theta}\int_0^{r^2} r^2\cos\theta \, dzdrd\theta$ 04

Question-6

(A) Change the order of integration $\int_0^4\int_y^4 \frac{x}{x^2+y^2} dxdy$ 04

(B) **Attempt any two.** 06

(a) Evaluate $\int_0^a\int_0^{\sqrt{a^2-y^2}} (x^2+y^2)\, dxdy$ by changing in to polar co-ordinates.

(b) Evaluate $\int_0^1\int_y^{1+y^2} x^2 y \, dxdy$

(c) Find the area lying between the parabola $y = x^2$ and the line $y^2 = x$

END OF PAPER