Date: 20|12|2016. Exam No:____________

GANPAT UNIVERSITY

B.TECH. SEM- I (All) (New) CBCS Regular Examination Dec-2016

2HS101: Calculus

Time: 3 HRS. Total Marks: 60

Instructions:

(1) This Question paper has two sections. Attempt each section in separate answer book.

(2) Figures on right indicate marks.

(3) Be precise and to the point in answering the descriptive questions.

SECTION: I

Question: 1

(A) Find n^{th} derivative of (i) $\sin x \cdot \sin 2x \cdot \sin 3x$ (ii) $\dfrac{x}{(x-1)(x-2)(x-3)}$ [4]

(B) Using Maclaurin's series expand $\tan x$ in power of x up to x^5. [3]

(C) Evaluate: (i) $\lim\limits_{x\to 0} \dfrac{2\sin x - \sin 2x}{x^3}$ (ii) $\lim\limits_{x\to 0} \left(\dfrac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$ [3]

OR

Question: 1

(A) Expand $\log x$ in power of $(x-1)$ up to fifth power and hence evaluate $\log 1.1$. [4]

(B) If $y = \sin\left(m \sin^{-1} x\right)$ prove that $(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} + (m^2-n^2)\, y_n = 0$ [3]

(C) Find n^{th} derivative of $\dfrac{x^4}{(x-1)(x-2)}$ [3]

Question: 2

(A) If $u = \log(\tan x + \tan y + \tan z)$ PT $\sin 2x \cdot u_x + \sin 2y \cdot u_y + \sin 2z \cdot u_z = 2$ [4]

(B) If $r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}\dfrac{y}{x}$ then find $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ [3]

(C) Examine x^3+y^3-3axy $(a>0)$ for extreme values. [3]

OR

Question: 2

(A) If $z = f(x,y)$, $u = lx+my$ & $v = ly - mx$ P.T. $z_{xx} + z_{yy} = (l^2+m^2)(z_{uu}+z_{vv})$ [5]

(B) Find minimum value of $x+y+z$ subject to the condition $\dfrac{4}{x}+\dfrac{9}{y}+\dfrac{16}{z} = 25$ [5]

Question: 3

(A) If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then P.T. $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{-\sin u \cos 2u}{4\cos^3 u}$ [4]

(B) Attempt any Two [6]

(1) If $u = f(x+at) + g(x-at)$ then P.T. $u_{tt} = a^2\, u_{xx}$

(2) Expand $x^4 - 11x^3 + 43x^2 - 60x + 14$ in power of $(x-3)$ up to fourth power.

(3) Find an approximate value of $f(x,y) = x^3+y^3$ when $x = 3.025$ and $y = 4.152$.

SECTION: II

Question: 4

(A) Derive reduction formula for $\tan x$ and hence evaluate $\int_0^{\frac{\pi}{4}} \tan^4\theta \, d\theta$. [4]

(B) Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$ using reduction formula [3]

(C) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz \, dx \, dy \, dz$. [3]

OR

Question: 4

(A) Evaluate $\int_0^{\pi} \sin^2\theta \; (1+\cos\theta)^4 \; d\theta$ using reduction formula [4]

(B) Derive reduction formula for $\cot\theta$ between $\frac{\pi}{4}$ to $\frac{\pi}{2}$ [3]

(C) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$. [3]

Question: 5

(A) Define Gamma function and prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ [4]

(B) Prove: (i) $erf\,(\infty) = 1$ (ii) $erf\,(-x) = -erf\,(x)$ [3]

(C) Define Beta function and hence evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ [3]

OR

Question: 5

(A) Derive the formula of relationship between beta and gamma function. [4]

(B) Define Complete elliptic integral of first kind and evaluate $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1+4\sin^2\theta}}$ [3]

(C) Evaluate $\int_0^{\infty} \frac{x^5}{5^x} \, dx$ in terms of gamma function. [3]

Question: 6

(A) Find $\iint_R xy\,(x+y) \, dx \, dy$ over the region R bounded by the curves $y = x^2$ & $y = x$ [4]

(B) **Attempt any Two** [6]

(1) Evaluate: $\int_0^1 \int_0^x e^{\frac{y}{x}} \, dx \, dy$

(2) Evaluate: $\int_0^{\infty} \int_0^x x \cdot e^{\frac{-x^2}{y}} \, dy \, dx$ by changing the order of integration.

(3) Evaluate: $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$ by changing into polar coordinates.

End of Paper