Date: 19/12/2016

Seat No. ______

GANPAT UNIVERSITY

B.Tech. Sem- II (NEW) CBCS REMEDIAL EXAMINATION, NOV-DEC 2016

SUBJECT: 2HS102 LINEAR ALGEBRA

Time: 3 hrs

Total marks: 60

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Que-1 Answer the following

(a) Find the Inverse of matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ 4

(b) Test for the consistency and solve the system of equations $\begin{matrix} x + y = 1 \\ 2x + 3y = 1 \\ 5x - y = 11 \end{matrix}$ 3

(c) Find the Rank of matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ 3

OR

Que-1 Answer the following.

(a) Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ 4

(b) Check whether the given vectors are L.D. or L.I.? If L.D. then find a relation between them $x_1 = (3,1,1), x_2 = (2,0,-1), x_3 = (4,2,1)$ 3

(c) Investigate for what values of λ and μ the system $\begin{cases} x + y + z = 6 \\ x + 2y + 3z = 10 \\ x + 2y + \lambda z = \mu \end{cases}$ 3

have (i) no solution (ii) a unique solution and (iii) an infinite solutions.

Que-2 Answer the following.

(a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ 4

(b) Show that matrix is $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$ skew symmetric. 3

(c) Is $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ a unitary matrix? Justify. 3

OR

Que-2 Answer the following.

(a) State the Caley Hamilton theorem and find A^3 for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ 4

(b) If $P = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ obtain matrix $(I - P)(I + P)^{-1}$ 3

(c) If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$ then show that $A^* A$ is a Hermitian matrix 3

Que-3 (a) Diagonalise the matrix $A = \begin{bmatrix} 7 & 3 \\ 2 & 5 \end{bmatrix}$ 4

(b) **Attempt any two** 6

I Define Hermitian , Skew – Hermitian and Unitary Matrices.

Ii Solve the system of linear equation $\begin{cases} x - 3y - 2z = 0 \\ 2x - y + 4z = 0 \\ x - 11y + 14z = 0 \end{cases}$

Iii If $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ then using Caley Hemilton theorem find the matrix $A^5 - 3A^4 + A^3 - 7A^2 + 5A + I$.

Section – II

Que-4 Answer the following.

(a) Show that the set $M = \left\{ \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} / a, b \in R \right\}$ is Vector space under the matrix addition defined by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and the scalar multiplication defined by $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$. 6

(b) Show that $A = \{(t, 2t, -3t)/t \in R\}$ is a subspace of R^3 but $B = \{(t^2, t, 2t)/t \in R\}$ is not a subspace of R^3. 4

OR

Que-4 Answer the following.

(a) Check whether the $V = \{(1, x)/x \in R\}$ is a vector space or not under the operations $(1, x_1) + (1, x_2) = (1, x_1 + x_2)$ and $\alpha((1, x_1) = (1, \alpha x_1)$ 6

(b) Define Subspace and Show that $V = \{(x, y)/x = 3y)$ is a subspace of R^2 4

Que-5 Answer the following.

(a) Find the range , rank, null space and nullity for $T(a, b) = (a + b, a - b, b)$ 4

(b) Check whethet the function $T: R^2 \to R^2$, $T(x, y) = (x + 1, y)$ are L. T or not ? 3

(c) Test the convergence of $\sum_{n=2}^{\infty} \frac{1}{n\log n}$ 3

OR

Que-5 Answer the following.

(a) Let $T: V_3 \to V_3$ defined as $T(l_1) = (l_1 - l_2)$,$T(l_2) = (2l_2 + l_3)$, $(T(l_3) = l_1 + l_2 + l_3)$ verify the rank nullity theorem. 4

(b) If $e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)$ then P. T. $S = \{ e_1, e_2, e_3\}$ is a basis of R^3. 3

(c) Test the convergence of $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + - - - - - -$. 3

Que-6 (a) Write the polynomial $V = t^2 + 4t - 3$ as a linear combination of the polynomials $e_1 = t^2 - 2t + 5$, $e_2 = 2t^2 - 3t$, $e_3 = t + 3$. 4

(b) **Attempt any two** 6

I Determine wheather the following spans the vector space R^3. $V_1 = (3,1,4)$, $V_2 = (2, -3, 5)$, $V_3 = (5, -2,9)$ and $V_4 = (1, 4, -1)$

Ii Check wheather the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n+3}{n^2+2}$ is convergent or not ?

Iii For whoich value of k will the vecto $= (1, k, 5)$ as a linear combination of vectors $x_1 = (1, -3,2)$ and $x_2 = (2, -1,1)$

END OF PAPER