Student Exam No:_____

GANPAT UNIVERSITY

B.TECH SEM. II (All Branches) ENGINEERING

CBCS REGULAR EXAMINATION MAY / JUNE-2013

SUBJECT:- 2HS102: Engineering Mathematics-II (All Branch)

TIME:-3 HOURS **TOTAL MARKS-70**

INSTRUCTIONS:- (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section-I

Que-1 Answer the following. **12**

(a) Prove that $\beta(m,n) = \dfrac{\lceil m \lceil n}{\lceil m+n}$.

(b) Define Error function and prove that $erf(x) + erf_c(x) = 1$.

(c) Evaluate using Beta and Gamma function $\int_0^1 \dfrac{dx}{\sqrt{1-x^4}}$.

OR

Que-1 Answer the following. **12**

(a) Define the Beta function and evaluate $\int_0^1 x^5 \left(1-x^3\right)^{10} dx$.

(b) Prove that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \beta(m+1, n+1)$

(c) Using Elliptic Integral evaluate $\int_0^{\pi/2} \dfrac{dx}{\sqrt{\cos x}}$.

Que-2 Answer the following.

(a) Evaluate: $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx$. **3**

(b) By changing the order of integration and evaluate the integral $\int_0^\infty \int_0^x x e^{-\frac{x^2}{y}} dy dx$. **4**

(c) Evaluate the triple integral $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz\, dy\, dx$. **4**

OR

Que-2 Answer the following.

(a) Prove that: $\iint_R xy\, dx dy = \dfrac{1}{24}$, Where R is the region given by $x \geq 0,\ y \geq 0,\ \& x+y \leq 1$. **3**

(b) Evaluate the given integral by changing into Polar co-ordinates, $\int_0^a \int_0^{\sqrt{a^2-y^2}} \left(x^2+y^2\right) dx\, dy$. **4**

(c) Evaluate the triple integral $\int_0^1 \int_0^{1-x} \int_0^{(x+y)^2} x\, dz\, dy\, dx$. **4**

Que-3 **Attempt any three.** 12

(a) Evaluate $\iint r^3\, dr\, d\theta$ over the area bounded between the circles $r = 2\cos\theta$ & $r = 4\cos\theta$.

(b) Evaluate using Reduction formula $\int_5^7 \sqrt{(x-3)(7-x)}\,dx$.

(c) Evaluate $\int_0^{\pi} (1+\cos\theta+\cos^2\theta+\cos^3\theta)\sin^3\theta\, d\theta$.

(d) Find the area between the Parabolas $y^2 = 4ax$ & $x^2 = 4ay$.

Section-II

Que-4 **Answer the following.** 12

(a) State De Moiver's theorem and Prove: $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$

(b) Solve the equation $x^7 + x^4 + x^3 + 1 = 0$ and find the product of all roots.

(c) Prove that i^i is real and find the value of $\sin\left(\log_e i^i\right)$.

OR

Que-4 **Answer the following.** 12

(a) Write the polar form of (1) $1+i$ (2) $1-\sqrt{3}i$

(b) Prove that $\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta} = -ie^{i\theta}\cot\frac{\theta}{2}$

(c) Expand $\sin^4\theta\cos^2\theta$ in the series of multiple of θ.

Que-5 **Answer the following.**

(a) **Find the differential equation of the family of curves** $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constant. 3

(b) Solve : $\frac{dy}{dx} = xy + x + y + 1$ 4

(c) Solve : $y^2dx + (xy + x^2)\,dy = 0$ 4

OR

Que-5 **Answer the following.**

(a) **Find the differential equation of the family of curves** $y = Ae^x + \frac{B}{e^x}$, where A and B are arbitrary constant. 3

(b) Solve : $\left[1+e^{\frac{x}{y}}\right]dx + e^{\frac{x}{y}}\left[1-\frac{x}{y}\right]dy = 0$ 4

(c) Solve : $x\log x\frac{dy}{dx} + y = 2\log x$ 4

Que-6 **Attempt any three.** 12

(a) If $i^{i^{i^{\cdots^{\infty}}}} = A + iB$ *prove that* $(a)\tan\frac{\pi A}{2} = \frac{B}{A}$, $(b)A^2 + B^2 = e^{-\pi B}$

(b) Separate $\tan^{-1}(x+iy)$ into Real and Imaginary parts.

(c) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = c^2$

(d) Trace the curve: $r^2 = a^2\cos 2\theta$

End of Paper