MOETING.

D: 12/05/14.

Seat No. ______

GANPAT UNIVERSITY

B. Tech. Sem. – II (ALL) Regular Exam. MAY – 2014

Sub : (2HS102) Engineering Mathematics – II

Time: 3 hrs | Total marks: 70

Instruction : (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section - I

QUE 1

(A) Define : Beta & Gamma function and prove that : $\beta(m,n) = \beta(n,m)$ (12)

(B) Define : Error function and prove that $\operatorname{erf_c}(x) + \operatorname{erf_c}(-x) = 2$

(C) Evaluate in terms of Elliptic integral : $\displaystyle\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}}$

OR

QUE 1

(A) Evaluate in terms of Gamma function : $\displaystyle\int_0^1 \frac{x}{\sqrt{1-x^5}}\, dx$ (12)

(B) Prove that : (i) $\operatorname{erf}(\infty) = 1$ (ii) $\operatorname{erf}(-x) = -\operatorname{erf}(x)$

(C) Evaluate in terms of Elliptic integral $\displaystyle\int_0^{\pi/2} \frac{dx}{\sqrt{(4-x^2)(9-x^2)}}$

QUE 2

(A) Using Reduction formula Evaluate : $\displaystyle\int_0^{\pi} (1-\cos\theta)^3\, d\theta$ (03)

(B) Using Reduction formula evaluate : $\displaystyle\int_0^1 x^5 \sin^{-1} x\, dx$ (04)

(C) Evaluate $\displaystyle\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z} (x+y+z)\; dx\, dy\, dz$. (04)

OR

QUE 2

(A) Using Reduction formula Evaluate : $\displaystyle\int_0^{\pi} \theta \sin^7\theta \cos^4\theta\; d\theta$ (03)

(B) If $I_n = \displaystyle\int_0^{\pi/3} \tan^n x\, dx$ then prove that $I_n + I_{n-2} = \dfrac{(\sqrt{3})^{n-1}}{n-1}$. (04)

(C) Evaluate : $\displaystyle\int_0^{\pi/2}\int_0^1\int_0^2 z\, r^2 \sin\theta\; dz\, dr\, d\theta$. (04)

QUE 3 **Attempt Any Three** **(12)**

(A) Evaluate : $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$

(B) Change the order of integration and evaluate : $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$

(C) Transform into polar coordinates and evaluate : $\int_0^a\int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dx\,dy$

(D) Find volume of the Ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Section – II

QUE 4 **(12)**

(A) Express the following complex number into polar form : (i) $1+i\sqrt{3}$ (ii) $-i$

(B) Solve the complex equation : $x^8+x^5-x^3-1=0$

(C) If $x+iy=\cosh(u+iv)$ then prove that : $\frac{x^2}{\cosh^2 u}+\frac{y^2}{\sinh^2 u}=1$ & $\frac{x^2}{\cos^2 v}-\frac{y^2}{\sin^2 v}=1$

OR

QUE 4 **(12)**

(A) If $\begin{cases} a=\text{cis}\,2\alpha \\ b=\text{cis}\,2\beta \\ c=\text{cis}\,2\gamma \\ d=\text{cis}\,2\delta \end{cases}$ then prove that (i) $\sqrt{\frac{ab}{c}}+\sqrt{\frac{c}{ab}}=2\cos(\alpha+\beta-\gamma)$

(ii) $\sqrt{\frac{ab}{cd}}+\sqrt{\frac{cd}{ab}}=2\cos(\alpha+\beta-\gamma-\delta)$

(B) Expand : $\sin^7\theta$ in terms of sines of multiples of θ.

(C) If α and β be the roots of $x^2-2x+4=0$ then prove that : $\alpha^n+\beta^n=2^n\cos\left(\frac{n\pi}{3}\right)$

QUE 5 (A) If $x_r=\text{cis}\left(\frac{\pi}{2^r}\right)$ then evaluate $\lim_{n\to\infty}[x_1 x_2 x_3 \cdots\cdots x_n]$ **(04)**

(B) Separate $\cos^{-1}\left(\frac{3i}{4}\right)$ in to real and imaginary parts . **(04)**

(C) Find the orthogonal trajectory of the family of circles : $x^2+y^2=a^2$. **(03)**

OR

QUE 5 (A) Prove that : $\left[1+i\sqrt{3}\right]^n+\left[1-i\sqrt{3}\right]^n = 2^{n+1}\cos\left(\frac{n\pi}{3}\right)$ **(04)**

(B) Prove that : $\log\left\{\frac{a+ib}{a-ib}\right\} = 2i\tan^{-1}\left(\frac{b}{a}\right)$. Hence evaluate $\tan\left[i\log\left\{\frac{a+ib}{a-ib}\right\}\right]$ **(04)**

(C) Find the orthogonal trajectory of the family of parabolas : $y=ax^2$. **(03)**

QUE 6 **Attempt Any Three** **(12)**

(A) Solve : $y\sqrt{1-x^2}\,dy+x\sqrt{1-y^2}\,dx=0$

(B) Solve : $(x^2-y^2)\,dx = 2xy\,dy$

(C) Solve : $\frac{dy}{dx}+\left(1+\frac{1}{x}\right)y=\frac{1}{x}$

(D) Solve : $\{x^2-4xy-2y^2\}\,dx+\{y^2-4xy-2x^2\}\,dy = 0$

END OF PAPER