Date: 13/05/2017

Exam No:___________

GANPAT UNIVERSITY

B.TECH. SEM- I CBCS (New) Regular & Remedial Examination **April - June 2017**

2HS101: Calculus

Time: 3 HRS. Total Marks: 60

Instructions:

(1) This Question paper has two sections. Attempt each section in separate answer book.

(2) Figures on right indicate marks.

(3) Be precise and to the point in answering the descriptive questions.

SECTION: I

Question: 1

(A) Expand $\tan x$ in power of x up to x^5. [4]

(B) Find n^{th} derivative of $y = \dfrac{3x-2}{(x+1)^2(x+3)}$ [3]

(C) Evaluate: $(i)\ \lim\limits_{x\to 0} \dfrac{x - \sin x}{x^3}$ $(ii)\ \lim\limits_{x\to 0} \left(\dfrac{1}{x}\right)^{\tan x}$ [3]

OR

Question: 1

(A) Find n^{th} derivative of $(i)\ y = \sin^3 x \cdot \cos^2 x$ $(ii)\ y = \dfrac{2x-1}{x^2-5x+6}$ [4]

(B) Expand $\sin x$ in power of $\left(x - \dfrac{\pi}{2}\right)$ up to fifth power and hence evaluate $\sin 91°$. [3]

(C) If $y = \left(\sin^{-1} x\right)^2$, PT $(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} - n^2 y_n = 0$ [3]

Question: 2

(A) If $u = \tan^{-1}\left(\dfrac{y}{x}\right)$ then prove that $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$ [4]

(B) If $u = x + y$ and $v = \dfrac{x}{x+y}$ then find $\dfrac{\partial(u,v)}{\partial(x,y)}$ [3]

(C) Examine $x^2 - 2xy + \dfrac{y^3}{3} - 3y$ for extreme values. [3]

OR

Question: 2

(A) If $\dfrac{4}{x} + \dfrac{9}{y} + \dfrac{16}{z} = 25$ obtain the values of x, y, z which makes $x + y + z$ maximum. [5]

(B) If $u = f(r)$ and $r^2 = x^2 + y^2$ PT $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r} f'(r)$ [5]

Question: 3

(A) If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$ then prove that $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial x} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$ [4]

(B) Attempt any Two [6]

(1) If $u = f(x-y, y-z, z-x)$ then prove that $u_x + u_y + u_z = 0$

(2) Find an approximate value of $f(x,y) = x^3 + y^3$ when $x = 3.025$ and $y = 4.152$.

(3) Expand $x^4 + 5x^3 - 3x^2 + 12x - 4$ in power of $(x-2)$ up to fourth power.

SECTION: II

Question: 4

(A) Evaluate $\int_0^1 x^5 \sin^{-1} x \, dx$ using reduction formula [4]

(B) Derive reduction formula for $\tan\theta$ between 0 to $\frac{\pi}{4}$ [3]

(C) Evaluate $\int_0^1 \int_y^1 \int_0^{1+x} x \, dz \, dx \, dy$. [3]

OR

Question: 4

(A) Evaluate $\int_0^{\pi} \sin^2\theta \, (1+\cos\theta)^4 \, d\theta$ using reduction formula [4]

(B) Derive reduction formula for $\cot x$ and hence evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^4\theta \, d\theta$. [3]

(C) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz \, dx \, dy \, dz$. [3]

Question: 5

(A) Prove: (i) $erf\,(\infty) = 1$ (ii) $erf\,(-x) = -erf\,(x)$ [4]

(B) Define Complete elliptic integral of first kind and evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1+3\sin^2 x}}$. [3]

(C) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$ in terms of gamma function. [3]

OR

Question: 5

(A) Derive the formula of relationship between beta and gamma function. [4]

(B) Define Beta function and hence evaluate $\int_0^1 x^5 (1-x^3)^{10} \, dx$ using Beta Function [3]

(C) Define Gamma function and prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ [3]

Question: 6

(A) Find $\iint_R y \, dx \, dy$ over the region R bounded by the curves $y^2 = 4x$ & $x^2 = 4y$ [4]

(B) **Attempt any Two** [6]

(1) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dx \, dy$ by changing the order of integration.

(2) Evaluate $\int_0^1 \int_0^1 \frac{dx \, dy}{\sqrt{(1-x^2)(1-y^2)}}$

(3) Integrate (x^2+y^2) by changing in to polar coordinates between 0 to 2 and 0 to $\sqrt{2x-x^2}$

END OF PAPER