Date: 11/05/2017

Seat No. ______

GANPAT UNIVERSITY

B.Tech. Sem-II CBCS (New) Regular & Remedial Theory, April-June 2017

SUBJECT: 2HS102 LINEAR ALGEBRA

Time: 3 hrs | Total marks: 60

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que-1 Answer the following

(a) Find the Inverse of matrix $A = \begin{bmatrix} 2 & 5 & 6 \\ 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$ using Gauss Jordan method. 4

(b) Investigate for what values of λ and μ the system $\begin{cases} x+y+z=6 \\ x+2y+3z=10 \\ x+2y+\lambda z=\mu \end{cases}$ have (i) no solution (ii) a unique solution and (iii) an infinite solutions. 3

(c) Find the Rank of matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$. 3

OR

Que-1 Answer the following.

(a) Find the Inverse of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gauss Jordan method. 4

(b) Check whether the given vectors are L.D. or L.I.? If L.D. then find a relation between them $x_1 = (1,-1,1)$, $x_2 = (2,1,1)$, $x_3 = (3,0,2,)$ 3

(c) Test for the consistency and solve the system of equations $\begin{matrix} x+y+z=6 \\ x-y+z=2 \\ 2x+y-z=1 \end{matrix}$ 3

Que-2 Answer the following.

(a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. 4

(b) Express matrix is $A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrices. 3

(c) If $A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ obtain matrix $(I-A)(I+A)^{-1}$. 3

OR

Que-2 Answer the following.

(a) Check whether the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is a diagonalizable or not? 4

(b) Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is an unitary matrix. 3

(c) If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$ then show that $A^{*}A$ is a Hermitian matrix. 3

Que-3 (a) Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ 4

(b) **Attempt any two** 6

i Define Hermitian, Skew – Hermitian and Unitary Matrices.

ii Solve the system of linear equation $\begin{cases} x - 3y - 2z = 0 \\ 2x - y + 4z = 0 \\ x - 11y + 14z = 0 \end{cases}$

iii If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 1 & 1 & -1 \end{bmatrix}$ then using Caley Hemilton theorem find A^{-1}.

Section – II

Que-4 **Answer the following.**

(a) Show that the set $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} / a, b, c, d \in R \right\}$ is Vector space under the usual matrix addition scalar multiplication. 6

(b) Show that $S = \{(X_1, X_2, X_3) / X_1 + 2X_2 = 0\}$ is a subspace of R^3. 4

OR

Que-4 **Answer the following.**

(a) Show that $V = R^2 = \{(x, y) / x, y \in R\}$ is a vector space under the usual vector addition and scalar multiplication. 6

(b) Check whether $S = \{(X_1, X_2, X_3) / X_1$ is an integer$\}$ is a subspace of R^3 or not? 4

Que-5 **Answer the following.**

(a) Is the vector $w = (2, -5, 3)$ a linear combination of vectors $v_1 = (1, -3, 2)$, $v_2 = (2, -4, -1)$, $v_3 = (1, -5, 7)$? 4

(b) Check whethet the function $T: M_{22} \to R^2, T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (ad + 1, b + c)$ are L. T or not ? 3

(c) Test the convergence of $\sum \frac{(n+1)^n x^n}{n^n + 1}$. 3

OR

Que-5 **Answer the following.**

(a) Check whethet the function $T: R^2 \to R^2$, $T(x, y) = (x + 1, y)$ are L. T or not ? 4

(b) Determine whether $(2, 5, -3)$ is in the span of $S = \{(2, -1, 1), (1, 1, -1), (4, 2, 1)\}$ or not ? 3

(c) Test the convergence of $\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + - - - -$. 3

Que-6 (a) Define $T: R^3 \to R^2$ by $T(X_1, X_2, X_3) = (X_1 - X_2, X_1 + X_3)$ then find $R(T)$ and $N(T)$. 4

(b) **Attempt any two** 6

i Show that $V_1 = (1, 1, -1), V_2 = (1, 0, 2), V_3 = (3, 2, 2)$ form a basis for R^3.

ii For whoich value of k will the vector $= (1, k, 5)$ as a linear combination of vectors $x_1 = (1, -3, 2)$ and $x_2 = (2, -1, 1)$.

iii Check wheather the alternating series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ is convergent or not?

END OF PAPER