Date: 25/05/2016

Exam No: ______________

GANPAT UNIVERSITY

B. Tech. Sem-I & II CBCS (NEW) (All Branches) Regular & Remedial Theory APRIL-JUNE 2016

Subject: 2HS101 CALCULUS

TIME: 3 HRS **TOTAL MARKS: 60**

Instructions: (1) This question paper has two sections. Attempt each section in separate answer book.
(2) Figures on right indicate marks.
(3) Be precise and to the point in answering the descriptive questions.

SECTION: I

Q.1 Answer the following

(a) Evaluate (I) $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$ (II) $\lim_{x \to 0} \sin x \tan x$ 4

(b) Find n^{th} derivative of $\frac{2x-3}{(x-1)(x-2)(x-3)}$. 3

(c) If $y = (x^2-1)^n$, prove that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$ 3

OR

Q.1 Answer the following

(a) Evaluate (I) $\lim_{x \to 1} \frac{1+\cos \pi x}{(x-1)^2}$ (II) $\lim_{x \to \infty} x^{1/x}$ 4

(b) If $y = \sin(ax+b)$, prove that $y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$. 3

(c) If $y = x\sqrt{x^2+1}$, prove that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-4)y_n = 0$ 3

Q.2 Answer the following

(a) If $u = \tan^{-1}(x^2+2y^2)$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u . \cos 3u$ 4

(b) If $u = e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2).e^{xyz}$ 3

(c) If $z = f(x,y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$. 3

OR

Q.2 Answer the following

(a) Show that the function $x^3 + y^3 - 63(x+y) + 12xy$ is maximum at $(-7,-7)$ and minimum at (3,3). 4

(b) If $x^x y^y z^z = c$, prove that $\frac{\partial^2 z}{\partial x \partial y} = -(x.\log_e ex)^{-1}$. 3

(c) Use Lagrange method of undetermined multipliers to find the shortest distance from the point (1,2,2) to the sphere $x^2+y^2+z^2 = 16$. 3

Q.3

(a) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{4}\right)$ up to fourth power, hence evaluate $\sin 46°$ up to four decimal places. 4

(b) Attempt any two

I Find n^{th} derivative of $e^{2x}\cos 3x \cos x$.

II If $v = \frac{1}{r}$ where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

III If $u = xyz, v = xy + yz + zx, w = x + y + z$, prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = (x-y)(y-z)(z-x)$

SECTION: II

Q.4 Answer the following

(a) Express the integral $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ in terms of gamma function. **4**

(b) Evaluate $\int_0^{\pi/6} \sin^2 6x \cos^3 3x \, dx$ using reduction formula. **3**

(c) Prove that $\beta(m,n) = \frac{(m-1)(n-1)}{(m+n-1)(m+n-2)}\beta(m-1, n-1)$. **3**

OR

Q.4 Answer the following

(a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$ then prove that $I_n + I_{n-2} = \frac{1}{n-1}$. Hence evaluate I_5. **4**

(b) Prove that $\int_0^{\infty} x\, e^{-x^8} dx \times \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi\sqrt{2}}{32}$ **3**

(c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{2-\cos 2x}}$ in terms of elliptic integral. **3**

Q.5 Answer the following

(a) Evaluate $\iint x \, dy \, dx$ over the region bounded between parabola $y = x^2$ and line $y = x + 6$. **4**

(b) Evaluate $\int_0^1 \int_y^{1+y^2} x^2 y \, dx \, dy$. **3**

(c) Evaluate the integral $\int_0^4 \int_y^4 \frac{x\, dx\, dy}{x^2+y^2}$ by changing order of integration. **3**

OR

Q.5 Answer the following

(a) Change to polar form and evaluate $\int_0^{a} \int_y^{a} \frac{x\, dx\, dy}{x^2+y^2}$. **4**

(b) Evaluate $\iint r^3 \, dr \, d\theta$ over the area bounded between the circles $r = 2\cos\theta$ & $r = 4\cos\theta$. **3**

(c) Find the area using double integration between parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **3**

Q.6

(a) Evaluate $\int_{-1}^{1} \int_0^{x} \int_{x-y}^{x+y} (z - 2x - y) \, dz \, dy \, dx$. **4**

(b) Attempt any two **6**

I Evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{1 + 3\sin^2 x} \, dx$ in term of elliptic integral.

II Prove that $\text{erf}(\infty) = 1$.

III Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z \, dz \, dy \, dx$.

END OF PAPER