Date: 11/06/16

GANPAT UNIVERSITY

B. Tech. Sem-I & II CBCS (NEW) (All Branches) Regular & Remedial Theory APRIL-JUNE 2016

Subject: 2HS102 LINEAR ALGEBRA

TIME: 3 HRS **TOTAL MARKS: 60**

Instructions: (1) This question paper has two sections. Attempt each section in separate answer book.
(2) Figures on right indicate marks.
(3) Be precise and to the point in answering the descriptive questions.

SECTION: I

Q.1 Answer the following

(a) Test for the consistency and if consistent solve the system $\begin{matrix} x-3y-8z=-10 \\ 2x+5y+6z=13 \\ 3x+y-4z=0 \end{matrix}$. 4

(b) Find inverse of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss Jordan method. 3

(c) Find rank of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ -6 & 3 & -9 \end{bmatrix}$. 3

OR

Q.1 Answer the following

(a) Find eigen value and eigen vector for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. 4

(b) Diagonalize the matrix $\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$. 3

(c) Check whether the following vectors are linearly dependent or linearly independent. If they are linearly dependent, find relation between them. (1,2,3), (2,−1,3) and (0,1,−1). 3

Q.2 Answer the following

(a) Using Caley-Hamilton theorem if $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, prove that 4

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}.$$

(b) Express $A=\begin{bmatrix} 5 & -2 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$ as sum of symmetric and skew-symmetric matrix. 3

(c) Solve the system $\begin{matrix} 2x+y-z=0 \\ 3x-2y-2z=0 \\ 12x-y-7z=0 \end{matrix}$. 3

OR

Q.2 Answer the following

(a) Solve the following system for x, y and z. 4

$-\frac{1}{x}+\frac{3}{y}+\frac{4}{z}=30,\ \frac{3}{x}+\frac{2}{y}-\frac{1}{z}=9$ and $\frac{2}{x}-\frac{1}{y}+\frac{2}{z}=10$.

(b) If $A=\begin{bmatrix} 1+i & 4 & -2+3i \\ -7 & -i & 3-4i \end{bmatrix}$, verify $A\cdot A^*$ is a Hermitian. 3

(c) Find conjugate transpose matrix of $\begin{bmatrix} 1-i & 2 & -2 \\ 0 & 1+i & -2+3i \end{bmatrix}$. 3

Q.3

(a) Verify Caley-Hamilton theorem and hence find A^{-1} where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

(b) **Attempt any two** 6

I Check whether the matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is Hermition or skew-Hermition?

II For what value of λ the system $x+y+z=1$

$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

have (1) no solution (2) unique solution (3) infinite solution.

III Is $A = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ unitary? Justify.

SECTION: II

Q.4 Answer the following

(a) Show that $M_{22} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} / a, b, c, d \in R \right\}$ is vector space under usual operations. 6

(b) Show that the span $\{(1,0,1), (-1,2,3), (0,1,-1)\}$ is all of R^3. 4

OR

Q.4 Answer the following

(a) Show that $p_1 = 1 + x - 3x^2, p_2 = 2x + x^2, p_3 = 3 + x$ forms a basis for P_2. 6

(b) Is vector $v = (6,11,6)$ in linear combination of $v_1 = (2,1,4), v_2 = (1,-1,3)$ and $v_3 = (3,2,5)$? Justify. 4

Q.5 Answer the following

(a) Check whether the following function is linear transformation or not? $T: R^2 \to R^2, T(x,y) = (x+y, x-y)$. 4

(b) Find range of the linear transformation $T: R^3 \to R^2, T(x,y,z) = (x+y, y+z)$. 3

(c) Test the convergence of $\sum_{n=1}^{\infty} \frac{2n^2+3n}{n^5+5}$. 3

OR

Q.5 Answer the following

(a) Verify rank-nullity theorem for $T: R^3 \to R^2, T(x,y,z) = (x+y, y)$. 4

(b) Define linear transformation and kernel of linear transformation. 3

(c) Test the convergence of $\sum_{n=1}^{\infty} \left(\frac{n+1}{n+2}\right)^n x^n$. 3

Q.6

(a) Determine inverse transformation of following linear transformation if exists. $T: R^2 \to R^2, T(x,y) = (x+y, y)$. 4

(b) **Attempt any two** 6

I Check whether the set $W = \{(x,y,z) | \ y = x + z + 1\}$ is subspace or not under usual operations?

II Test the convergence of alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$.

III Test the convergence of the series $\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots$.

END OF PAPER