Seat No.:______

GANPAT UNIVERSITY

B.TECH. SEM.- III [B.M.&I.] EXAMINATION

NOVEMBER – DECEMBER ; 2010.

SUB: MATHEMATICS-II

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Question-1 Attempt any three. **(12)**

(a) If $f(x) = x + x^2$, $-\pi < x < \pi$, prove that

$$f(x) = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right) + 2\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right).$$

(b) Find a Fourier series to represent if $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$. Hence deduce that: $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} - \cdots\cdots\cdots$

(c) Find the half range Cosine series for: $f(x) = (x-1)^2$; $0 < x < 1$

(d) Obtain Fourier series of $f(x) = e^{2x}$, $-\pi \le x \le \pi$

Question-2 Attempt any three: **(12)**

(a) Define Laplace transform and using definition find $L\{f(t)\}$ if :

$$f(t) = \begin{cases} 2, & 0 \le t < 4 \\ 1, & t > 4 \end{cases}$$

(b) Find: **(1).** $L\left\{e^{-2t}(2t+1)\right\}$ **(2).** $L^{-1}\left\{\frac{s+1}{s^2 - 2s + 3}\right\}$

(c) If $L\{f(t)\} = \bar{f}(s)$, prove that $L\{t.f(t)\} = -\frac{d\bar{f}(s)}{ds}$. Hence evaluate $L\{t.\sin 2t\}$

(d) Using Laplace transform method solve: $\frac{d^2y}{dt^2} + y = t$; Where $y(0) = 0$, $y'(0) = 1$.

Question-3 (A) Attempt the following: **(8)**

(a) Express the function $f(x) = \begin{cases} -e^{2x} & x < 0 \\ e^{-2x} & x > 0 \end{cases}$ as Fourier integral and hence prove that

$$\int_0^\infty \frac{\lambda.\sin \lambda x}{\lambda^2 + 2^2} d\lambda = \frac{\pi}{2} e^{-2x} \quad if\ x > 0$$

(b) Express e^{-x}, $x \ge 0$ as Fourier sine transform and hence deduce that

$$\int_0^\infty \frac{x.\sin mx}{1 + x^2} dx = \frac{\pi}{2} e^{-m}$$

(B) State Convolution theorem and using it find $L^{-1}\left\{\frac{s}{(s-3)(s^2+1)}\right\}$ **(3)**

Section – II

4 (A) Prove that $\sinh z$ is an analytic function of the variable $z = x + iy$. **(03)**

(B) **Attempt any two:** **(08)**

(1) Find the analytic function whose real part is $x^2 - y^2$.

(2) Find the Bilinear transformation which maps the points $z = -1,\ i,\ 1$ onto the points $w = 1,\ i,\ -1$.

(3) Prove that $\int_c \frac{dz}{z-a} dz = 2\pi i$; where c is the circle $|z-a| = r$

5 **Attempt any three:** **(12)**

(A) State and Prove Cauchy's theorem.

(B) Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)} dz$ where c is the circle $|z| = 3$.

(C) Evaluate $\int_0^6 \frac{dx}{1+x^2} dx$ with h=1 by using Trapezoidal rule.

(D) Find the real root of $f(x) = x^3 - 3x - 5 = 0$ between 2 and 3 by Bisection method correct up to two decimal places.

6 **Attempt any three:** **(12)**

(A) Find the real root of $f(x) = x^3 - 2x - 5 = 0$ between 2 and 3 by false position method correct up to three decimal places.

(B) Find $(701)^{\frac{1}{3}}$ correct up to two decimal places by Newton-Raphson method.

(C) Obtain Picard's Second approximate solution of $\frac{dy}{dx} = x - y^2$ for x = 0.1correct up to four decimal places with $y(0) = 1$.

(D) Apply Runge-kutta method to find an approximate value of y at x = 0.1 with h = 0.1 given that $\frac{dy}{dx} = x + y^2$ and y = 1 when x = 0.