Seat No.______

GANPAT UNIVERSITY
B.TECH.SEM.III (BM & I) EXAMINATION. N0V/DEC - 2011
Sub : 2HS301- Mathematics-III

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que-1 **(12)**

(a) $If\ L\{f(t)\} = \overline{f}(s)\ then\ Prove\ that\ L\left\{\int_0^t f(u)\,du\right\} = \frac{\overline{f}(s)}{s}$

(b) $Find\ (1)\ L\{e^{-t}\sin^2 t\}\quad (2)\ L\left\{\frac{1-e^{-2t}}{t}\right\}$

(c) $Find\ L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$

OR

Que-1 **(12)**

(a) $Find\ L^{-1}\left\{\frac{s}{s^4+s^2+1}\right\}$

(b) $Find\ (1)\ L\{e^t u(t-3)\}\quad (2)\ L\left\{t\int_0^t e^t\, sint\, dt\right\}$

(c) State Convolution theorem and apply it to evaluate $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$

Que-2

(a) $Find\ a\ Fourier\ series\ for\ the\ function\ \ f(x) = x^2\ ;\ [-\pi, \pi]$ **(03)**

$Hence\ show\ that\quad \frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} - \cdots\ldots\ldots\ldots$

(b) *Obtain a Fourier series for the* f^{ns} $f(x)$ *defined as* **(04)**

$$f(x) = \begin{cases} \frac{\pi}{2} + x \;;\; -\pi \le x \le 0 \\ \frac{\pi}{2} - x \;;\; 0 \le x \le \pi \end{cases}$$

(c) *Find a Fourier series for the* f^{ns} $f(x) = \sqrt{1 - cosx}\,;\; 0 < x < 2\pi$ **(04)**

OR

Que-2

(a) *Expand* $f(x) = e^{-x}$ *as a Fourier series for* $-l < x < l$ **(03)**

(b) *Find the fourier expansion of* $f(x) = 1 - t^2 \;;\; -1 \le x \le 1$ **(04)**

(c) *Obtain a Fourier series for the* f^{ns} $f(x)$ *defined as* **(04)**

$$f(x) = \begin{cases} 1 + x \;;\; -1 \le x \le 0 \\ 1 - x \;;\; 0 \le x \le 1 \end{cases}$$

Que-3 **(12)**

(a) *Solve* : $\frac{d^2y}{dt^2} + 4y = sint\,;$ *where* $y(0) = 1\,, y'(0) = 0$

(b) *Expand* $f(x) = \pi x - x^2$ *as a Half − range sine series in* $0 < x < \pi$

(c) *Find the Fourier sine transform of* e^{-IxI}

Hence evaluate $\int_0^{\infty} \frac{x\, sinmx}{1 + x^2}\, dx$

Section – II

(12)

(a) Find an analytic function whose imaginary part is $e^x\left[x\cos y - y\sin y\right]$.

(b) Find bilinear transformation which maps the points $z = 1, i, -1$ on to $w = 0, 1, \infty$.

(c) Evaluate $\int_0^{1+i} \left(x^2 + iy\right) dz$ along the path $y = x^2$

OR **(12)**

Que-4

(a) Evaluate $\int_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where c is the circle $c : |z| = 3$

(b) Prove that the function $\sinh z$ is an analytic function.

(c) Show that the function $u = y + e^x \cos y$ is harmonic and find its conjugate.

Que-5

(a) Evaluate $\int_0^1 \frac{dx}{1+x}$ by (i) Trapezoidal rule (ii) Simpson's one-third rule, dividing the range into six equal parts, **(04)**

(b) Apply Gauss-Jordan method to solve: **(04)**
$3x + y + 2z = 3,\ 2x - 3y - z = -3, x - 2y + z = -4$.

(c) Find Picard's third approximate solution of $\frac{dy}{dx} = x + y$; $y(0) = 1$ **(03)**

OR

Que-5

(a) By Gauss-elimination method solve: **(04)**
$x + 2y - z = 3,\ 3x - y + 2z = 1, 2x - 2y + 3z = 2$.

(b) Apply Runge-Kutta method to find an approximate value of y for x=1.4 given that **(04)**
$\frac{dy}{dx} = xy$; $y(1) = 2$ *taking* $h = 0.2$

(c) Use Euler's method to solve $y' = x + y^2$; $y(0) = 1$ to find y at x=0.5 **(03)**

Que-6 **Attempt any three** **(12)**

(a) State and Prove Cauchy's Residue theorem.

(b) Expand $f(z) = \frac{1}{(z-2)(z-1)}$ in Laurent series valid for $|z| < 1$, $1 < |z| < 2$

(c) Find a real root of the equation $e^{3x} = 3x$ between 0 & 1 correct up to three decimal places using Newton-Raphson method.

(d) Find a real root of the equation $x^3 - 3x - 5 = 0$ between 2 & 3 correct up to three decimal places using Bisection method.

END OF PAPER