Seat No.______

GANPAT UNIVERSITY

B.Tech. (BM&I) Sem – III CBCS Regular Theory Examination

Subject: 2HS301: Mathematics – III

November – December 2012

Time: - 3 Hours **Total Marks: 70**

Instructions:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION – I

Que-1 **Answer the following.** **(12)**

[A] Prove that $Sin\ hz$ is an analytic function.

[B] If $f(z) = u + iv$ is an analytic function of z then find $f(z)$ where $u - v = e^x(Cosy - Siny)$

[C] Show that $u = y^3 - 3x^2y$ is a Harmonic function. Find its Harmonic conjugate. Also find corresponding analytic function.

OR

Que-1 **Answer the following.** **(12)**

[A] Prove that the function $\frac{1}{z}$ is an analytic function.

[B] Find an analytic function whose real part is $\frac{Sin2x}{(Cosh2y - Cos2x)}$

[C] If $f(z)$ is an analytic function with constant modulus; then show that $f(z)$ is constant.

Que-2 **Answer the following.**

[A] State & Prove Cauchy's theorem. **(03)**

[B] Evaluate: $\int_0^{2+i} (\bar{z})^2 dz$; along the real axis to 2 and vertically to $2+i$. **(04)**

[C] Evaluate : $\int_c \frac{e^{2z}}{(z-1)(z-2)} dz$; where c is the circle: $|z| = 3$. **(04)**

OR

Que-2 **Answer the following.**

[A] Prove that $\int_c \frac{dz}{z-a} = 2\pi i$; where c is the circle $|z-a| = r$ **(03)**

[B] Evaluate: $\int_0^{1+i} (x^2 + iy) dz$; along the path $y = x^2$. **(04)**

[C] Evaluate : $\int_c \frac{3z^2+z}{z^2-1}\,dz$; where c is the circle $|z-1|=1$

Que-3 **Attempt any three:** **(12)**

[A] Expand: $f(z)=\frac{1}{z}$ as a Taylors series about the point $z=1$

[B] Show that $\frac{1}{4z-z^2}=\sum_{n=0}^{\infty}\frac{z^{n-1}}{4^{n+1}}$; when $0<|z|<4$.

[C] Expand: $\frac{1}{z(z^2-3z+2)}$ for the regions (1) $0<|z|<1$ and (2) $1<|z|<2$.

[D] Find the Bilinear transformation which maps the points $z=2, i, -2$ in to the points $w=1, i, -1$

SECTION – II

Que-4 **Answer the following.** **(12)**

[A] Find a Fourier series to represent the function $f(x)=x^2;\ -\pi\le x\le\pi$. Hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\ldots=\frac{\pi^2}{6}$

[B] Find a Fourier series to represent the function *f(x)* is given by $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$

[C] Find a half rang sine series for $f(x)=1;\ 0\le x\le 2$

OR

Que-4 **Answer the following.** **(12)**

[A] Find a Fourier series to represent the function $f(x)=e^x;\ 0\le x\le 1$

[B] Find a Fourier series to represent the function f(x) is given by $f(x)=\begin{cases}-x, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$

[C] Find the Fourier series for $f(x)=-\pi x^3,\ -\pi<x<\pi$

Que-5 **Answer the following.**

[A] Evaluate: $L^{-1}\left\{\frac{s}{s^4+s^1+1}\right\}$ **(03)**

[B] Evaluate: (1) $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$ (2) $L\left\{t^2\sin^2 t\right\}$ **(04)**

[C] By using Laplace transform evaluate $\int_0^\infty e^{-2t} t \sin 4t\,dt$ **(04)**

OR

Que-5 **Answer the following.**

[A] Evaluate: $L\left\{ te^{-4t} \sin 3t \right\}$. **(03)**

[B] Evaluate: (1) $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$ (2) $L\left\{\frac{1-\cos t}{t}\right\}$ **(04)**

[C] Solve differential equation $y''' + 2y'' - y' - 2y = 0$, $y(0) = 1$, $y'(0) = 2$, $y''(0) = 2$ **(04)**

Que-6 **Attempt any three:** **(12)**

[A] Obtain Picard's second approximate solution of the initial value problem $\frac{dy}{dx} = x^2 + y^2$ for x = 0.4, given that y (0) = 0.

[B] Find an approximate value of $\sqrt{17}$, correct up to three decimal places, using Newton's Raphson method.

[C] State convolution theorem and evaluat: $L^{-1}\left\{\frac{1}{(s+1)(s+3)}\right\}$.

[D] Apply Gauss-Jacobi's method to solve: $6x - y - z = 19$, $3x + 4y + z = 26$, $x + 2y + 6z = 22$.

END OF PAPER