Seat No.______

GANPAT UNIVERSITY
B.Tech. (B.M.&I.) Sem-III
SUBJECT: 2HS301 Mathematics – III-Theory
CBCS Regular Examination Nov-Dec 2013.

TIME: - 3 HOURS **TOTAL MARKS: 70**

INSTRUCTIONS:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION – I

Que-1 Answer the following. (12)

[A] Determine the function $\frac{x}{x^2+y^2}+i\frac{y}{x^2+y^2}$ is an analytic function or not.

[B] If $f(z)=u+iv$ is an analytic function of z then find $f(z)$ where $u-v=\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^{y}-e^{-y}}$

[C] If $f(z)$ is an analytic function of z then prove that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]|f(z)|^2=4|f'(z)|^2$

OR

Que-1 Answer the following. (12)

[A] Prove that the function $\frac{x-iy}{x^2+y^2}$ is an analytic function.

[B] Find an analytic function $w=u+iv$; given that $v=\frac{x}{x^2+y^2}+\cosh x\cos y$

[C] If $f(z)$ is an analytic function with constant modulus; then show that $f(z)$ is constant.

Que-2 Answer the following.

[A] State & Prove Cauchy's theorem. (03)

[B] Evaluate: $\int_0^{2+i}\left(\overline{Z}\right)^2 dz$; along the real axis to 2 and the vertically to $2+i$. (04)

[C] Evaluate: $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$; where c is the circle: $|z|=3$. (04)

OR

Que-2 Answer the following.

[A] Prove that $\int_c \frac{dz}{z-a}=2\pi i$; where c is the circle $|z-a|=r$. (03)

[B] Evaluate: $\int_0^{1+i}\left(x^2+iy\right)dz$; along the path $y=x^2$. (04)

[C] Evaluate : $\int_c \frac{e^{2z}}{(z+1)^4}dz$;where c is the circle $|z|=2$. (04)

Que-3 Attempt any three: **(12)**

[A] Expand: $f(z)=\frac{1}{z}$ as a Taylor's series about the point $z=1$.

[B] Find the Bilinear transformation which maps the points $z=1,i,-1$ in to the points $w=0,1,\infty$ respectively.

[C] Expand: $\frac{1}{z(z^2-3z+2)}$ for the regions (1) $0<|z|<1$ and (2) $1<|z|<2$.

[D] Find the Bilinear transformation which maps the points $z=1,i,-1$ in to the points $w=i,0,-i$ respectively.

SECTION – II

Que.-4 Attempt the following: **(12)**

(A) Find a real root of equation $2x-\log_{10}x=7$ using self iteration method correct to four decimal places.

(B) Using convolution theorem, evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.

(C) Expand $f(x)$ as Fourier series in the interval $-\pi\le x\le\pi$ if $f(x)=e^{ax}$

OR

(A) Apply Jacobi's iteration method to solve the equations

$20x+y-2z=17,\ 3x+20y-z=-18,\ 2x-3y+20z=25$

(B) If $L\{f(t)\}=\bar{f}(s)$ then prove that $L\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}\bar{f}(s), n=1,2,3,\ldots$

(C) Find the Fourier series of $f(x)=\begin{cases}\pi x & ,0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$

Que.-5 Attempt the following: **(11)**

(A) Given that **(4)**

x	1	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.75	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=1.6$

(B) Evaluate $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$ **(4)**

(C) Find Fourier series of $f(x)=x\sin x$ in $-\pi\le x\le\pi$ **(3)**

OR

(A) Use Simpson's 3/8[th] rule to find $\int_0^6\frac{1}{1+x^2}dx$ where step size is 1. **(4)**

(B) Solve $y''+y=t$ by Laplace transform method where $y(\pi)=0, y'(0)=1$ **(4)**

(C) Obtain a Fourier series for $f(x)=\begin{cases}-1, & -\pi<x<0\\ 1, & 0<x<\pi\end{cases}$ **(3)**

Que.-6 Attempt any three: **(12)**

(A) Find the Fourier integral representation of function $f(x)=\begin{cases}1, & |x|<1\\ 0, & |x|>1\end{cases}$

(B) Find the Fourier integral representation of function $f(x)=\begin{cases}-e^{kx}, & x<0\\ e^{-kx}, & x>0\end{cases}$

(C) Evaluate (1) $L\left\{\frac{\cos at-\cos bt}{t}\right\}$ (2) $L\{t\,e^{2t}\cos 3t\}$

(D) Find half range Fourier cosine series $f(x)=\begin{cases}1, & 0\le x\le 1\\ x, & 1\le x\le 2\end{cases}$

END OF PAPER