Seat No. ________________

Ganpat University

B. Tech Semester – III (BM&I)Regular Examination Nov – Dec – 2014

Subject : (2HS 301) Mathematic – III

Time: 3 hrs. **Marks: 70**

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

Section – I

Que: 1

(A) If $L\{f(t)\} = \overline{f}(s)$ then Prove that $L\left\{\int_0^t f(u)\,du\right\} = \dfrac{\overline{f}(s)}{s}$ [4]

(B) Find : (1) $L\{e^{-3t}(\cos 4t + \sin 4t)\}$ (2) $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$ [4]

(C) Find $L^{-1}\left\{\log\left(\dfrac{s-2}{s+3}\right)\right\}$ [4]

OR

Que: 1

(A) Find (1) $L\{t\cos at\}$ (2) $L^{-1}\left\{\dfrac{s+7}{s^2+2s+2}\right\}$ [4]

(B) State Convolution theorem and apply it to evaluate $L^{-1}\left\{\dfrac{1}{s^2(s-1)}\right\}$ [4]

(C) Solve : $\dfrac{d^2y}{dt^2} + 4y = \sin t$; where $y(0) = 1, y'(0) = 0$ [4]

Que: 2

(A) Find a Fourier series for the function $f(x) = x + x^2$; $[-\pi, \pi]$ [3]

Hence show that $\dfrac{\pi^2}{6} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots\ldots\ldots$

(B) Obtain a Fourier series for the f^{ns} $f(x)$ defined as [4]

$$f(x) = \begin{cases} -x & ; \ -\pi \le x \le 0 \\ x & ; \ 0 \le x \le \pi \end{cases}$$

(C) Find a Fourier series for the f^{ns} $f(x) = 1 + \sin x$; $[-1, 1]$ [4]

OR

Que: 2

(A) Expand $f(x) = e^{-x}$ as a Fourier series for : $-l < x < l$ [3]

(B) Find the fourier expansion of $f(x) = x^2 - 2$; $-2 \le x \le 2$ [4]

(C) Obtain the Half range Cosine series for the f^{ns} $f(x)$ defined as [4]

$$f(x) = \begin{cases} 0 \; ; & 0 \le x \le \pi/2 \\ \pi/2 \; ; & \pi/2 \le x \le \pi \end{cases}$$

Hence show that $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots\ldots$

Que: 3 Attempt any Three

(A) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 \; ; & |x| < 1 \\ 0 \; ; & |x| > 1 \end{cases}$ [4]

(B) Find the Fourier Sine transform of : $f(x) = \begin{cases} 1 \; ; & 0 < x < a \\ 0 \; ; & x > a \end{cases}$ [4]

(C) Find : $L^{-1}\left\{\frac{2s^2 - 4}{(s+1)(s-2)(s-3)}\right\}$ [4]

(D) Obtain a Fourier series for the f^{ns} $f(x)$ defined as [4]

$$f(x) = \begin{cases} 1 + x \; ; & -1 \le x \le 0 \\ 1 - x \; ; & 0 \le x \le 1 \end{cases}$$

Section – II

Que: 4

(A) Check the analyticity of (i) $f(z) = \bar{z}$ (ii) $f(z) = e^z$ [4]

(B) If $w = T_1(z) = \frac{z-2}{z+3}$ & $w = T_2(z) = \frac{z}{z+2}$ then find $T_1^{-1}, T_2^{-1}, T_1 \cdot T_2$ & $T_2 \cdot T_1$ [4]

(C) Determine the analytic function whose real part is $\cos x \cdot \cosh y$ [4]

OR

Que: 4

(A) Evaluate $\oint_C \frac{\cos \pi z + \sin \pi z}{(z-1)(z-2)} dz$ where C: $|z| = 3$ [4]

(B) Evaluate $\int_C [(x+y)dx + x^2 y \, dy]$ along $y = 3x$ between $(0,0)$ and $(3,9)$ [4]

(C) State and prove Cauchy's theorem for contour integration. [4]

Que: 5

(A) Obtain Laurent's series for $f(z) = \frac{1}{(z+3)(z+1)}$ in (i) $|z| < 1$ (ii) $1 < |z| < 3$ [4]

(B) Calculate $\left(\frac{dy}{dx}\right)_{x=1.5}$ for the following data. [4]

x	1	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

(C) Solve: $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$ by Gauss elimination method. [3]

OR

Que: 5

(A) State and prove Cauchy Residue theorem and find it for $f(z) = \frac{z^2 + 1}{z^2 - 2z}$ [6]

(B) State and prove maximum modulus thorem. [5]

Que: 6 Attempt any Three

(A) Using Bisection method find real root of $x^3 - 4x - 9 = 0$ in $(2, 3)$ upto fourth approximation [4]

(B) Apply False position Method to find real root of $x^3 - 2x - 5 = 0$ in $(2, 3)$ correct up to two decimal places. [4]

(C) Evaluate $\sqrt{28}$ in $(5, 6)$ correct upto three decimal places using N – R Method. [4]

(D) Evalute $\int_0^1 x^2\, dx$; with $h = 0.2$ using Simpson's one – third rule. [4]

End of Paper