Exam No. ______________

GANPAT UNIVERSITY

B. Tech. Semester – III (BM) CBCS (NEW) Examination, Nov. – 2016

Sub : (2HS304) Mathematics for Biomedical Engineering

Time : 03 Hrs **Total Marks : 60**

Instructions : (1) All questions are compulsory.

(2) Write answer of each sections in separate answer books.

(3) Figures to the right indicate marks of questions.

SECTION – I

Que – 1

(A) Evaluate : (i) $L\{e^t \cos 3t \cos 2t\}$ (ii) $L\{t \cos^2 t\}$ **(04)**

(B) If $L\{f(t)\} = \overline{f}(s)$ then Prove that $L\left\{\int_0^t f(u)\,du\right\} = \frac{\overline{f}(s)}{s}$. **(03)**

(C) Using Convolution theorem ; evaluate $L^{-1}\left\{\frac{1}{(s+7)(s-2)}\right\}$. **(03)**

OR

Que – 1

(A) Evaluate : (i) $L\left\{\frac{1-e^{2t}}{t}\right\}$ (ii) $L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\}$ **(04)**

(B) Evaluate : $L^{-1}\left\{\log\left(\frac{s+2}{s-5}\right)\right\}$ **(03)**

(C) Define Unit step Function . Express the given function in terms of unit step function and hence obtain its Laplace transform : $f(t) = \begin{cases} 0 \; ; \; t < 4 \\ t^2 \; ; \; t \geq 4 \end{cases}$ **(03)**

Que – 2

(A) Find a Fourier series for the function $f(x) = \pi^2 - x^2$; $[-\pi, \pi]$ **(04)**

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\cdots\cdots$

(B) Find the fourier expansion of $f(x) = \begin{cases} 0 \; ; \; -1 \leq x \leq 0 \\ x \; ; \; 0 \leq x \leq 1 \end{cases}$ **(03)**

(C) Expand $f(x) = e^{ax}$ as a Fourier series in $[-\pi, \pi]$. **(03)**

OR

Que – 2

(A) Find the fourier expansion of $f(x) = \begin{cases} x & ; 0 \le x \le 1 \\ 1-x & ; 1 \le x \le 2 \end{cases}$ **(04)**

(B) Obtain Half range Cosine series for $f(x) = \begin{cases} 0 & ; 0 \le x \le \pi/2 \\ \pi/2 & ; \pi/2 \le x \le \pi \end{cases}$ **(03)**

Hence show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\cdots\cdots$

(C) Find a Fourier series for the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 \le x \le 2\pi$. **(03)**

Que – 3

(A) Find the Fourier transform of : $f(x) = \begin{cases} 1 & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$. **(05)**

Hence evaluate : $\int_0^\infty \frac{\sin x}{x} dx$.

(B) Use transform method to solve : $\frac{d^2y}{dt^2} + 4y = 0$; **(05)**

where $y(0) = 1$, $y'(0) = 6$.

OR

(B) Evaluate : $L^{-1}\left\{\frac{1}{(s+1)(s^2+2s+2)}\right\}$ **(05)**

SECTION – II

Que – 4

(A) If $f(z) = u + iv$ is an Analytic function of z then find f(z) If **(04)**

$u - v = (x - y)(x^2 + 4xy + y^2)$

(B) Evaluate : $\int_0^{1+i} (x - y + i x^2)\, dz$; along **(03)**

(i) the straight line from $z = 0$ to $z = 1 + i$ and

(ii) the real axis from $z = 0$ to $z = 1$ & then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

(C) State and Prove : Liouvilles theorem **(03)**

OR

Que – 4

(A) Show that $u(x,y) = e^{-2xy}\sin(x^2 - y^2)$ is harmonic function and find a harmonic conjugate of $u(x,y)$. **(04)**

(B) If f(z) is Analytic function of z then Prove that : **(03)**

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4\,|f'(z)|^2 .$$

(C) Evaluate : $\int_C \frac{e^z}{(z+1)^2}\,dz$; where C is the circle : $|z-1| = 3$. **(03)**

Que – 5

(A) Apply Gauss – seidel iteration method to solve the equations : **(04)**

$10x + y + z = 12 , 2x + 10y + z = 13 , 2x + 2y + 10z = 14$.

(B) Find the real root of $x\sin x + \cos x = 0$ correct upto four decimal places by using Newton – Raphson method . **(03)**

(C) Find the real root of $xe^x - 3 = 0$ by False position method up to three decimal places . **(03)**

OR

Que – 5

(A) Evaluate : $\int_0^6 \frac{dx}{1+x^2}$ By Trapezoidal and simpsons' 3/8 rules . **(04)**

(B) Find the real root of $x^3 - x - 11 = 0$ upto two decimal places by using the Bisection method . **(03)**

(C) Apply Gauss – elimination method to solve the equations : **(03)**

$2x + y + z = 10 , 3x + 2y + 3z = 18 , x + 4y + 9z = 16$.

Que – 6 Attempt any two **(10)**

(A) Using Picards process of successive approximations obtain a solution upto the fifth approximation of the equation : $\frac{dy}{dx} = x + y$; $y(0) = 1$.

(B) Using Eulers method find an approximate value of y corresponding to $x = 1.0$; given that $\frac{dy}{dx} = x + y$; $y(0) = 1$.

(C) Apply 4^{th} order R – K method to find approximate value of y at $x = 0.2$ given that : $\frac{dy}{dx} = \sqrt{x+y}$; $y(0) = 1$.

END OF PAPER