Seat No.______

GANPAT UNIVERSITY

B.Tech. (C.E/I.T.) Sem-III CBCS Regular Theory Examination.

SUBJECT: 2HS301 Mathematics - III

Nov-Dec 2012.

TOTAL MARKS: 70

TIME: - 3 HOURS

INSTRUCTIONS:

1.All questions are compulsory.
2.Write answer of each section in separate answer books.
3.Figures to the right indicate marks of questions.

SECTION – I

Question-1 Attempt the following: (12)

(A) Evaluate : (1) $L\{e^{-t}\sin 3t\cos 2t\}$ (2) $L\{t.\cosh t\}$

(B) Evaluate: (1) $L^{-1}\left\{\dfrac{2s+1}{s^2+2s+6}\right\}$ (2) $L^{-1}\left\{\dfrac{1}{(S+1)(S^2+1)}\right\}$

(C) Using Laplace method, solve the initial value problem $\dfrac{d^2y}{dt^2}-3\dfrac{dy}{dt}+2y=0$, where $y(0)=1$, $y'(0)=-1$

OR

Question-1 (12)

(A) Find the Laplace Transform of $f(t)=\begin{cases}\sin t, & 0<t<\pi \\ 0, & t>\pi\end{cases}$

(B) If $L\{f(t)\}=\overline{f(s)}$, prove that $L\left\{\dfrac{f(t)}{t}\right\}=\displaystyle\int_s^\infty \overline{f(s)}\,ds$. Using it find $L\left\{\dfrac{\sinh t}{t}\right\}$

(C) State convolution theorem and apply it to evaluate: $L^{-1}\left\{\dfrac{1}{(S^2+a^2)^2}\right\}$

Question-2 Attempt the following:

(A) Expand : $f(x)=\sin x$ in a half range cosine series in the interval $(0,\pi)$ (03)

(B) Find a Fourier series for the function : $f(x)=\begin{cases}x & ; \quad 0\le x\le\pi \\ 2\pi-x & ; \quad \pi\le x\le 2\pi\end{cases}$ (04)

(C) Find a Fourier series to represent : $f(x)=x^2$, $-\pi<x<\pi$ (04)

Hence deduce : $\dfrac{\pi^2}{12}=\dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+---$

OR

Question-2

(A) Find the half range cosine series to represent $f(x)=1-\dfrac{x}{2}$, $0\le x\le 1$ (03)

(B) Find a Fourier series to represent the function $f(x)=x+|x|$, $-\pi\le x\le\pi$ (04)

(C) Find a Fourier series for the function: $f(x)=\dfrac{(\pi-x)^2}{4}$, $0<x<2\pi$ (04)

Question-3 Attempt any three: **(12)**

(A) Express the function $f(x)=\begin{cases} -e^{kx} & ; x<0 \\ e^{-kx} & ; x>0 \end{cases}$ as Fourier integral and hence prove that $\int_0^\infty \frac{\lambda \sin\lambda x}{\lambda^2+k^2}\,d\lambda = \frac{\pi}{2}e^{-kx}\ ;\ x>0, k>0$

(B) Find Fourier transform of $f(x)=\begin{cases} k & ; 0<x<a \\ 0 & ; \text{otherwise} \end{cases}$

(C) Using partial fraction method, find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$

(D) (1) Find : $L^{-1}\left\{\log\left(\frac{s+1}{s-1}\right)\right\}$ (2) Prove that: $L\{\sin at\}=\frac{a}{s^2+a^2};s>0$

SECTION-II

Question-4 Attempt the following: **(12)**

(A) If imaginary part of an analytic function is $e^{-x}(x\cos y + y\sin y)$ then find real part.

(B) Find the bilinear transformation which maps the points 2,i,-2 in to the points 1,i,-1.

(C) Evaluate $\oint_C \frac{\cos\pi z^2}{(z-1)(z-2)}dz$ where C is the circle $|z|=3$

OR

(A) If $f(z)=u+iv$ is an analytic function of z then find $f(z)$ if $u-v=(x-y)(x^2+4xy+y^2)$

(B) Find the bilinear transformation which maps the unit circle $|z|=1$ in to the real axis in such a way that the points $z=1,i,-1$ are mapped into the points $w=0,1,\infty$ respectively.

(C) Find the value of $\int_0^{2+i}(\bar{z})^2\,dz$ along the real axis from 0 to 2 and then vertically from 2 to 2+i.

Question-5 Attempt the following: **(12)**

(A) Prove that $(i)E=e^{hD}$ $(ii)E\nabla=\Delta$

(B) Find value of y when $x=110$ from the following observation table

x	100	150	200	250	300	350	400
y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

(C) Find root of equation $\cos x = xe^x$ using Newton Raphson method correct up to three decimal places.

OR

(A) Given $y_{35.0}=1175, y_{35.5}=1280, y_{39.5}=2180, y_{40.5}=2420$ find y_{40} by Lagrange's interpolation formula.

(B) Find a real root of equation $\cos x = 3x-1$ upto three decimal places by iteration method.

(C) Use Euler's method to solve $y' = x+y^2$ where $y(0)=1$and $h=0.1$ to find $y(0.5)$.

Question-6 Attempt the following:

(A) From the difference table of $f(x)=x^3-3x^2+5x+7$ for the values of x =0,2,4,6,8 and extend the table for the calculation of $f(x)$ for $x=10$. **(4)**

(B) Use Gauss seidal method to solve $83x+11y-4z=95$, **(4)**

$$7x+52y+13z=104, 3x+8y+29z=71$$

(C) Evaluate $\int_4^{5.2}\log_e x\,dx$with $h=0.2$ using trapezoidal rule. **(3)**