Seat No. ______

Ganpat University
B.Tech Semester-III (CE, IT) CBCS Regular & Remedial Theory examination
Subject: 2HS301 - Mathematics - III
December 2013

TIME: - 3 HOURS **TOTAL MARKS: 70**

INSTRUCTIONS:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION – I

Question-1 Attempt the following: **(12)**

(A) Define Laplace transform and using it show that $L\{sinat\} = \frac{a}{s^2+a^2}$

(B) Evaluate: (1) $L\{e^t \cos^2 t.\sin^2 2t\}$ (2) $L\{t.cosat\}$

(C) Using partial fraction ,find $L^{-1}\left\{\frac{2s-1}{(s+1)(s-2)}\right\}$

Question-1 **OR** **(12)**

(A) Using definition of Laplace transform, show that $L\{t^n\} = \frac{\overline{)n+1}}{s^{n+1}}$.

(B) Define periodic function. Using it find Laplace transform of Half wave rectifier $f(t) = \begin{cases} sin\omega t & ; \quad 0 < t < \pi/\omega \\ 0 & ; \quad \pi/\omega < t < \pi \end{cases}$

(C) Evaluate: (1) $L\left\{\int_0^t sint\, dt\right\}$ (2) $L\left\{\frac{1-e^t}{t}\right\}$

Question-2 Attempt the following:

(A) Show that Fourier series of function $f(x) = x^3 \quad -\pi \le x \le \pi$ is given by **(04)**

$$f(x) = 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left[-\pi^2+\frac{6}{n^2}\right] sinnx$$

(B) Find a Fourier series expansion of the function $f(x) = e^{ax}, -\pi \le x \le \pi$. **(04)**

(C) Obtain Half range cosine series for $f(x) = lx - x^2 \; ; \; 0 \le x \le l$ **(03)**

Question-2 **OR**

(A) Find a Fourier series for the function : $f(x) = \begin{cases} \pi & ; \quad 0 < x < \pi \\ \pi - x & ; \quad \pi < x < 2\pi \end{cases}$ **(04)**

(B) Find a Fourier series expansion of the function $f(x) = x + x^2, -\pi \le x \le \pi$. **(04)**

(C) Obtain Half range cosine series for $f(x) = e^x \; ; \; 0 \le x \le \pi$ **(03)**

Question-3 Attempt any three: **(12)**

(A) Find the Fourier integral representation of $f(x) = \begin{cases} 1 & ; \quad |x| < 1 \\ 0 & ; \quad |x| > 1 \end{cases}$ and hence evaluate $\int_0^{\infty}\frac{sin\lambda.cos\lambda x}{\lambda}\,d\lambda$

(B) Using Laplace transform method solve: $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=0$, Where $y(0)=1,\ y'(0)=0$

(C) State and apply convolution theorem to evaluate: $L^{-1}\left\{\frac{1}{S(S^2+1)}\right\}$

(D) Show that: $\int_0^\infty \frac{\cos\lambda x}{1+\lambda^2}\,d\lambda = \frac{\pi}{2}e^{-x};\ x \ge 0$

Section-II

Question-4 (12)

(A) Determine a, b, c and d so that the function $f(z)=(x^2+axy+by^2)+i(cx^2+dxy+y^2)$ is analytic.

(B) Discuss the analyticity of $f(z)=\sinh z$

(C) Find the image of infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. Also show the region graphically.

Question -4 **OR** (12)

(A) Show that $u=y+e^x\cos y$ is harmonic function and determine their conjugate.

(B) Evaluate : $\int_c (z-z^2)\,dz$ where c is the upper half of the circle $|z|=1$

(C) Find the Bilinear transformation which maps the points $z=-1,1,\infty$ in to $w=-i,-1,i$.

Question -5

(A) Find an approximate value of $\sqrt{17}$, correct up to three decimal places, using Newton's Raphson method (03)

(B) Evaluate: $\int_0^5 \frac{dx}{4x+5}$ using Trapezoidal rule by dividing the interval into 10 equal parts (04)

(C) Find root of $\frac{dy}{dx}=x+y^2$; y(0) = 1 using Euler's method at x = 0.5. Take step length h=0.1 (04)

Question -5 **OR**

(A) Using Taylor's series method obtain y at x = 0.1 if $\frac{dy}{dx}=3x+y^2$, given that y (0) = 1. (03)

(B) Using the following data estimate Y when X =8. (04)

X	0	5	10	15	20	25
Y	7	11	14	18	24	32

(C) Apply Gauss-Seidal method to solve : $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.5$. (04)

Question-6 : Attempt any three: (12)

(A) Using following table evaluate f(9) by Lagrange's interpolation formula.

X	5	7	11	13	17
Y	150	392	1452	2366	5202

(B) Use Runge-Kutta forth order method to find y at x=0.2 for the I.V.P $\frac{dy}{dx}=x+y,\ y(0)=1$

(C) Evaluate : $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$ where c is the circle $|z|=3$

(D) Form the difference equation from the relation $y_n = A.2^n + B.3^n$ where A and B are arbitrary constant.

End of Paper