Seat No. _____

GANPAT UNIVERSITY

B. TECH. SEM. III (CE/IT) EXAMINATION. NOV/DEC – 2014

Sub : (2HS301) Mathematics – III

Time: 3 hrs **Total marks: 70**

Instructions: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que-1 **(12)**

(A) Find a root of the equation : $x^3 - 4x - 9 = 0$ using Bisection method up to two decimal places.

(B) Find a real root of the equation : $x e^x = \cos x$ by False – Position method up to three decimal places.

(C) Apply Gauss – Ellimination method to solve : $\begin{cases} x - y + z = 6 \\ 3x + 2y - 2z = -2 \\ 2x + 4y + z = 3 \end{cases}$

OR

Que-1 **(12)**

(A) Using Newton Raphson method Find a real root of the equation : $x e^x - 2 = 0$ up to three decimal places.

(B) Apply Gauss – Seidal method to solve : $\begin{cases} 10x + y + z = 12 \\ 2x + 10y + z = 13 \\ 2x + 2y + 10z = 14 \end{cases}$

(C) Using Taylor's series method find y at $x = 0.1$ up to five decimal places from $\frac{dy}{dx} = x - y^2$; $y(0) = 1$

Que-2

(A) Using Picards process of successive approximations obtain a solution upto third approximation of the equation : $\frac{dy}{dx} = x - y^2$; $y(0) = 0$. **(03)**

(B) Apply 4^{th} order R – K method to find approximate value of y at $x = 0.2$ given that ; $\frac{dy}{dx} = x + y$; $y(0) = 1$. **(04)**

(C) The following table gives some relations between Steam pressure (P) and Temperature (T). Find the pressure at temp. 372° by Lagranges interpolation formula **(04)**

T	361°	367°	378°	387°	399°
P	154.9	167.0	191.0	212.5	244.2

OR

Que-2

(A) Using Eulers method find an approximate value of y corresponding to $x = 0.1$; Given that $\frac{dy}{dx} = \frac{y-x}{y+x}$; $y(0) = 1$. (04)

(B) Evaluate : $\int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal & Simpson's 1/3 rule. (04)

(C) Given ; $\begin{cases} \log_{10} 100 = 2 \\ \log_{10} 101 = 2.0043 \end{cases}$ and $\begin{cases} \log_{10} 103 = 2.0128 \\ \log_{10} 104 = 2.0170 \end{cases}$

Evaluate $\log_{10} 102$ By Newtons formula .

Que-3 **Attempt Any Three** (12)

(A) Determine analytic function whose real part is : $x^2 - y^2$.

(B) Find the bilinear transformation which maps the points $Z = 2, i, -2$ onto the points $W = 1, i, -1$.

(C) Evaluate : $\int_0^{1+i} (x^2 + iy)\, dz$; Along (1) the line $y = x$ and (2) $y = x^2$

(D) Evaluate : $\int_C \frac{e^{2z}}{(z-1)(z-2)}\, dz$; where C is the circle : $|z| = 3$.

Section – II

Que-4 (12)

(A) If $L\{f(t)\} = \bar{f}(s)$ then Prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)\, ds$

(B) Find $L\{f(t)\}$ for $f(t) = \begin{cases} \sin t \ ; & t < \pi \\ t \ ; & t \geq \pi \end{cases}$.

(C) Evaluate $L^{-1}\left\{\frac{1+2s}{(s-1)^2(s+2)^2}\right\}$

OR

Que-4 (12)

(A) Find the Laplace transform of the periodic triangular wave function

$f(t) = \begin{cases} t \ ; & 0 < t < a \\ 2a - t \ ; & a < t < 2a \end{cases}$

(B) Solve $4y'' - 4y' + 37y = 0$ by Laplace transform method where $y(0) = 3, y'(0) = 1.5$.

(C) Evaluate $L^{-1}\left[\tan^{-1}\left(\frac{2}{s^2}\right)\right]$

Que-5

(A) Show that $0 \leq x \leq \pi$ the cosine series for $x(\pi - x)$ is $\frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \cdots\right)$ (4)

(B) Obtain a Fourier series for $f(x) = \begin{cases} l - x \ ; & 0 < x < l \\ 0 \ ; & l \leq x \leq 2l \end{cases}$ (4)

(C) Obtain a Fourier series of $f(x) = |\sin x|$ for $-\pi < x < \pi$ (3)

OR

Que-5

(A) Obtain Fourier series for $f(x) = \begin{cases} x + \frac{\pi}{2} & ; -\pi < x < 0 \\ \frac{\pi}{2} - x & ; 0 < x < \pi \end{cases}$ **(4)**

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\cdots$

(B) Find the Fourier series expansion of $f(x) = \frac{(\pi - x)^2}{4}$; $0 < x < 2\pi$. **(4)**

Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

(C) Prove that in interval $0 < x < \pi$, $\frac{e^{ax} - e^{-ax}}{e^{a\pi} - e^{-a\pi}} = \frac{2}{\pi}\left(\frac{\sin x}{a^2+1} - \frac{2\sin 2x}{a^2+4} + \frac{3\sin 3x}{a^2+9} - \cdots\right)$ **(3)**

Que-6 **Attempt Any Three** **(12)**

(A) Find Fourier sine integral representation of function $f(x) = \begin{cases} \sin x & ; 0 \le x \le \pi \\ 0 & ; x > \pi \end{cases}$

(B) Find the Fourier integral representation of function $f(x) = \begin{cases} -e^{kx} & ; x < 0 \\ e^{-kx} & ; x > 0 \end{cases}$.

Hence deduce that $\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2 + K^2)} d\lambda = \frac{\pi}{2} e^{-kx}$; if $x > 0, k > 0$

(C) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$.

(D) Find half range Fourier cosine series $f(x) = \begin{cases} 1 & ; 0 \le x \le 1 \\ x & ; 1 \le x \le 2 \end{cases}$

End Of Paper