

TIME: - 3 Hours

Total Marks: 60

**Instructions:**

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

**SECTION - I****Question-1 Attempt the following:**

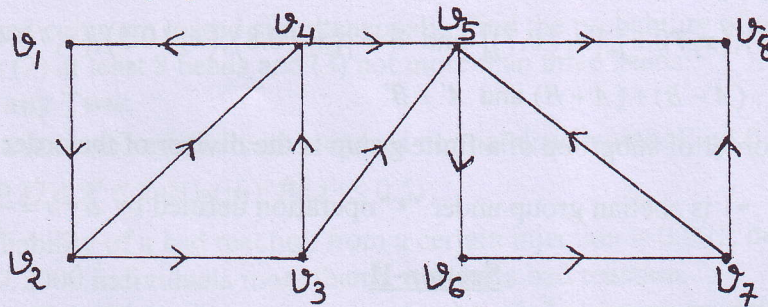
- (A) Prove that: (04)
- (1) In a group, inverse of each element is unique.
  - (2) Every cyclic group is abelian.
- (B) Show that the set  $G = \{1, -1, i, -i\}$  is cyclic group under multiplication operation. Also find all possible generators of it. (03)
- (C) In the set  $R - \{1\}$  define an operation "\*" as  $a * b = a + b - ab, \forall a, b \in R - \{1\}$ . Show that  $\langle R - \{1\}, * \rangle$  is group. (03)

**Question-1****OR**

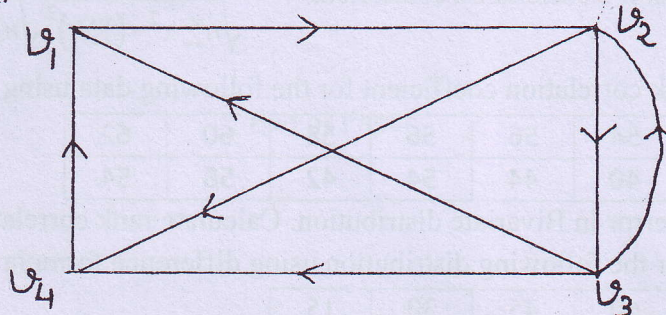
- (A) Attempt the following : (04)
- (1) State and prove left cancellation law in group.
  - (2) State and prove reversal law of inverse in group.
- (B) Let  $X = \left\{ \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} / \alpha \in R \right\}$ . Prove that  $\langle X, \cdot \rangle$  where ' $\cdot$ ' is usual matrix multiplication operation. (03)
- (C) Define Left and Right cosets. If  $H = \{0, 3\}$  and  $a = 2$ , find  $aH$  and  $Ha$  (03)

**Question-2 Attempt the following:**

- (A) Is the graph given below weakly connected, strongly connected or unilaterally connected? (04)  
Justify answer with proper reason.



- (B) Define Adjacency matrix representation of directed multigraph. Also find it for the following graph. (03)



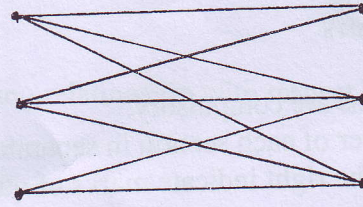
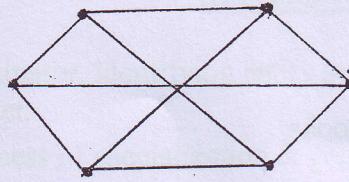
- (C) Define following terms and draw suitable graph to explain them : (03)
- (1) Reachability
  - (2) Out degree of vertex
  - (3) Path of graph.



OR


(A)

Define Isomorphic graph. Check whether the following graphs are isomorphic or not. Give reason(s). (04)



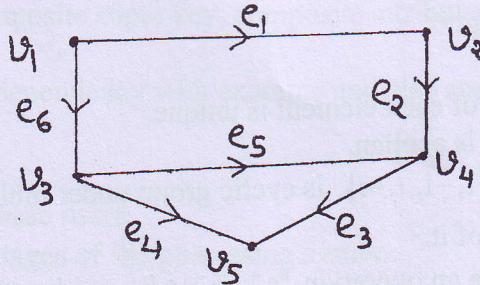
(B)

Define Incident matrix representation for simple di-graph. Also find it for the following graph.



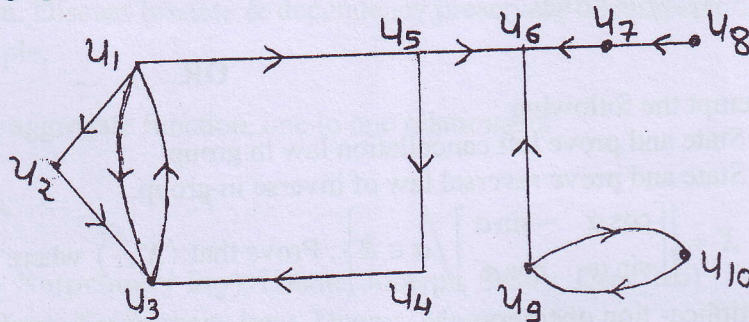
```

graph LR
    1 ---|e1| 2
  
```



(C)

Define Node base of graph. Find all possible Node base of the following graph.



**Question-3 Attempt the following:**

(A)

Verify (a)  $(A \cup B)' = A' \cap B'$  (b)  $(A \cap B)' = A' \cup B'$  if  $A = \{(x_1/0.3), (x_2/0.7), (x_3/0.2)\}$  and  $B = \{(x_1/0.1), (x_2/0.6), (x_3/1.0)\}$

(B) Attempt any Two

(a) If  $A = \{(x_1/0.3), (x_2/0.6), (x_3/0.7)\}$  and  $B = \{(x_1/0.8), (x_2/0.5), (x_3/0.4)\}$  find  $A - B$ ,  $A + B$ ,  $(A - B) + (A + B)$  and  $A' + B'$

(b) Prove that the order of subgroup of a finite group is the divisor of the order of group.

(c) Show that  $\langle Q^+, * \rangle$  is abelian group under "\*" operation defined by  $a * b = \frac{ab}{5}, \forall a, b \in Q^+$

## Section-II

**Question-4 Attempt the following:**

(A)

**Attempt the following:**

Derive Karl Pearson's correlation coefficient  $r = \frac{n\sum xy - \sum x \cdot \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$  (04)

(B)

Calculate the rank correlation coefficient for the following data using Spearman's formula (03)

$x$	46	54	56	56	58	60	62
$y$	36	40	44	54	42	58	54

(C)

y	36	40	44	54	42	58	54
---	----	----	----	----	----	----	----

Define probable error in Bivariate distribution. Calculate rank correlation coefficient and probable error for the following distribution using difference formula (03)

$x$	75	60	45	30	15
$y$	150	175	200	225	250



- (A) Derive the Spearman's rank correlation coefficient formula  $\rho = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$  (04)
- (B) Prove that correlation coefficient is independent of shift origin and change of scale. (03)
- (C) A computer while calculating correlation coefficient between  $x$  and  $y$  from 25 pairs of observations obtained the results  $\sum x = 125$ ,  $\sum y = 100$ ,  $\sum x^2 = 650$ ,  $\sum y^2 = 460$ ,  $\sum xy = 508$ . It was later discovered that two pairs of  $x$  and  $y$  values namely  $(x, y) = (6, 14)$ ,  $(x, y) = (8, 6)$  were entered wrongly while the correct values were  $(x, y) = (6, 14)$ ,  $(x, y) = (8, 6)$ . Obtain the correct value of correlation coefficient. (03)

**Question -5 Attempt the following:**

- (A) Derive the equation of regression line of  $y$  on  $x$  of the form  $y - \bar{y} = b_{yx}(x - \bar{x})$  (04)
- (B) Using difference formula compute regression line of  $x$  on  $y$  from the following data. (03)
- |     |     |     |     |     |     |    |     |    |    |    |
|-----|-----|-----|-----|-----|-----|----|-----|----|----|----|
| $x$ | 105 | 104 | 102 | 101 | 100 | 99 | 98  | 96 | 93 | 92 |
| $y$ | 101 | 103 | 100 | 98  | 95  | 96 | 104 | 92 | 97 | 94 |
- (C) Show that : (03)
- (1) Regression coefficients are independent of shift origin
  - (2) Arithmetic mean of regression coefficients is greater than correlation coefficient.

**Question -5**

OR

- (A) Using square formula compute regression line of  $y$  on  $x$  from the following data. (04)
- |     |    |    |    |    |    |    |    |    |    |    |
|-----|----|----|----|----|----|----|----|----|----|----|
| $x$ | 23 | 27 | 28 | 29 | 30 | 31 | 33 | 35 | 36 | 39 |
| $y$ | 18 | 22 | 23 | 24 | 25 | 26 | 28 | 29 | 30 | 32 |
- (B) Obtain the equation of regression line of  $x$  on  $y$  of the form  $x - \bar{x} = b_{xy}(y - \bar{y})$  (03)
- (C) Show that : (03)
- (1) Geometric mean between regression coefficients is equal to correlation coefficient.
  - (2) If a one regression coefficient is greater than unity then the other must be less than unity.

**Question-6**

- (A) 10 unbiased coins are tossed simultaneously. Find the probability of obtaining (1) exactly six heads (2) at least 8 heads and (3) not more than three heads. (04)
- (B) **Attempt any Two:** (06)
- (a) Let  $X$  be a random variable with standard normal distribution. Find (i)  $P(-0.73 \leq X \leq 0)$  (ii)  $P(-1.37 \leq X \leq 2.01)$  (iii)  $P(|X| \leq 0.5)$ .
- (b) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than 2 will get a bad reaction.
- (c) A quiz has 10 multiple choice questions. Each with 3 alternatives inclusive of the correct answer. Find the probability that the student get 8 or more correct answers.

End of Paper