Seat No. _____

GANPAT UNIVERSITY

B. TECH. SEM. IV [CE / IT] EXAMINATION. May/JUNE – 2012

Sub : (2HS401/CE305/IT305) Discrete Mathematics

Time : 3 hrs **Total marks : 70**

Instruction : (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Que 1 **(12)**

(*A*) Consider the set Z of integers and for integer $m > 1$ Define a relation on Z as :

$x \cong y \pmod m$ iff $(x - y)$ is divisible by m.

Then show that Z is an equivalence relation

(*B*) Let $\langle L, \leq \rangle$ be a lattice then for $a, b c \in L$ Prove that :

$$b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

(*C*) Let X be a non – empty set then show that $\langle p(X), \cap, \cup, \emptyset, X \rangle$ is a Complemented lattice .

OR

Que 1 **(12)**

(*A*) Let A be a set then prove that $\langle P(A), \subseteq \rangle$ is a Po – set.

(*B*) Let $\langle L, \leq \rangle$ be a lattice then for any $a, b, c \in L$ Prove That

$a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

(*C*) If $L = \{0,1\}$ Then Prove That $\langle L^2, \leq \rangle$ and $\langle L^3, \leq \rangle$ are Complemented lattices.

Que 2

(*A*) Define Boolean algebra and give any three examples of it with detail. **(03)**

(*B*) In a Boolean algebra B ; P.T. $(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$. **(04)**

(C) Write the following Boolean expression in an equivalent sum – of – Products canonical form in three variables A, B, C : (1) $A' \oplus (B' * C)$ (2) $A \oplus B$ **(04)**

OR

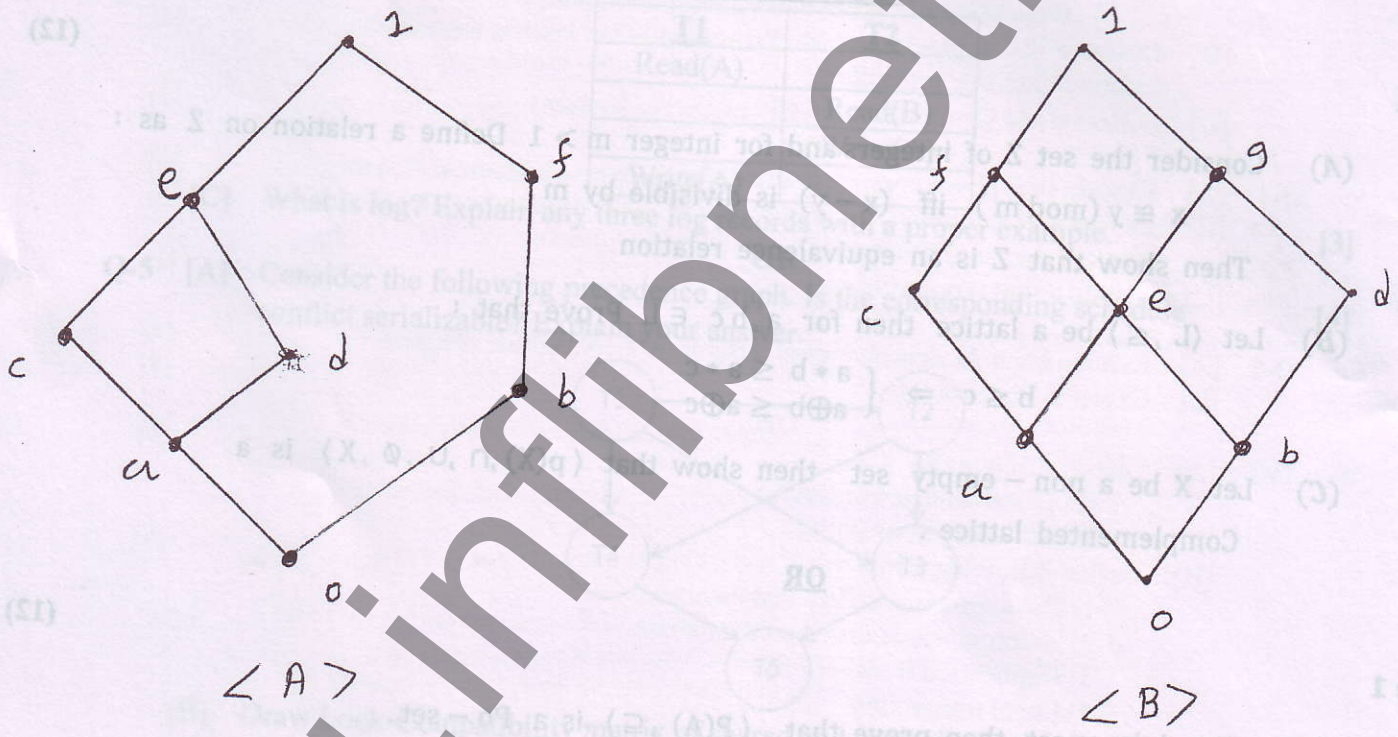
Que 2

(A) P.T. Any Boolean algebra is isomorphic to a power set algebra $\langle P(s), \cap, \cup, \sim, \emptyset, S \rangle$ **(03)**

(B) In a Boolean algebra B ; P.T. (1) $a \oplus (a * b) = a$ (2) $a' \oplus (a * b) = a' \oplus b$ **(04)**

(C) Define : (1) Join Irreducible element (2) Atoms of a Boolean algebra. **(04)**

Find the Join irreducible element and Atoms for the following figure.



Que 3 **Attempt any three :** **(12)**

(A) Explain : Predicates in Detail.

(B) Show that : $(x)\ (H(x) \rightarrow M(x)) \wedge H(s) \Rightarrow M(s)$.

(C) Prove that Every chain is a Distributive lattice .

(D) Let $f : B_1 \rightarrow B_2$ be an isomorphism of Boolean algebras. Prove that If x is an atom of B_1 then f(x) is an atom in B_2

Section – II

Que 4 Answer the Following **(12)**

(a) Define Group. Prove that $Z = \{0, 1, 2, 3, 4, 5, 6\}$ is commutative group under "Addition modulo 7"

(b) Define Normal sub group.

Prove that a subgroup N of a group G is a normal sub group of G iff ${}_gN_{g^{-1}} = N$

(c) Define order of group and order of an element.

Find order of each element of Group $G = \{I, A, B, C\}$ under multiplication.

Where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

OR

Que 4 Answer the Following. **(12)**

(a) Let R_2 be the set of all 2×2 real matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d \in R$ and $ad - bc \neq 0$. Prove that R_2 is Abelian group under the matrix multiplication.

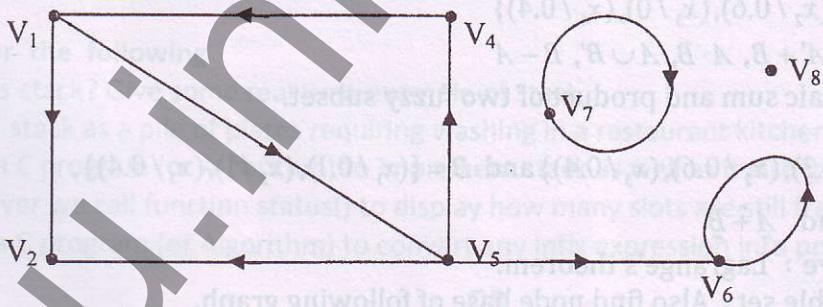
(b) Prove that $\langle G, * \rangle$, where $G = \{1, -1, i, -i\}$ and '$*$' is multiplication operation ; is cyclic group. Also find order of each its elements.

(c) Define Cyclic Group. Find all the sub group of a cyclic group of order 12 with generator 'a'. Also find order of each element and the other generator.

Que 5 Answer the Following.

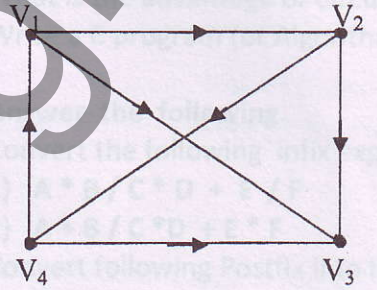
(a) Define : (1) Node (2) Adjacent node (3) Simple graph. **(03)**

(b) Define In – degree and Out – degree of vertex. Find In – degree and Out – degree of each node of following digraph. **(04)**

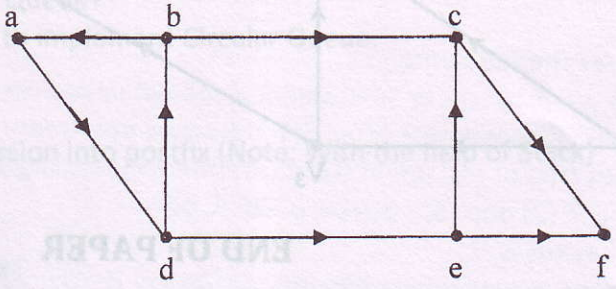


(c) Define : Weakly connected graph, Unilaterally connected graph and Strongly connected graph **(04)**

Check whether the given graphs are Weakly, Unilaterally or Strongly connected.



(a)



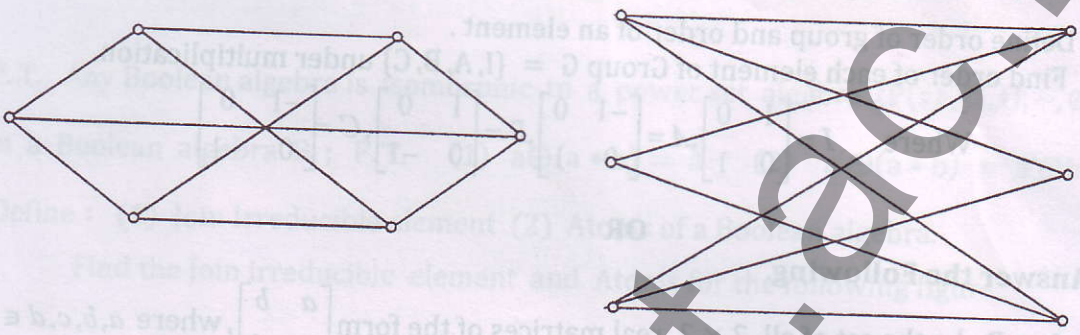
(b)

OR

Que 5 **Answer the Following.**

(a) Define: (1) Isolated Vertex (2) Odd vertex & Even vertex (3) Loop **(03)**

(b) Define: Isomorphic graphs. Check whether the given graphs are isomorphic or not ; Give reason. **(04)**



(c) Define: Adjacency matrix of a graph. Draw a di – graph for a given adjacency matrix **(04)**

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Que 6 **Attempt any three :** **(12)**

(a) If $A = \{(x_1 / 0.5), (x_2 / 0.3), (x_3 / 0.8), (x_4 / 1)\}$

$B = \{(x_1 / 0.8), (x_2 / 0.6), (x_3 / 0), (x_4 / 0.4)\}$

than find $A' + B,\ A \cdot B,\ A \cup B',\ B - A'$

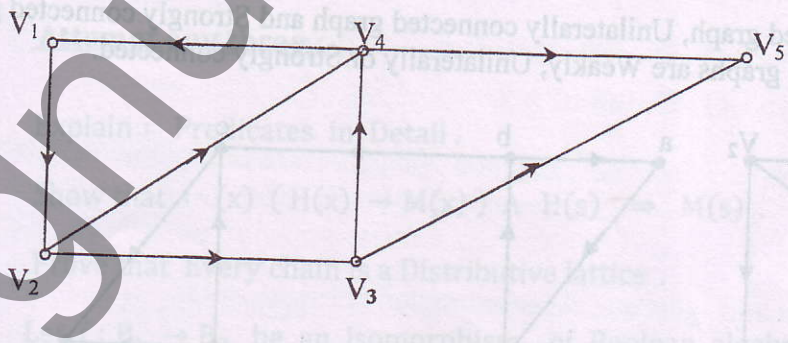
(b) Define algebraic sum and product of two fuzzy subset.

If $A = \{(x_1 / 0.3), (x_2 / 0.6), (x_3 / 0.4)\}$ and $B = \{(x_1 / 0.1), (x_2 / 1), (x_3 / 0.4)\}$,

Find $\underset{\sim}{A} \cdot \underset{\sim}{B}$ and $\underset{\sim}{A} \hat{+} \underset{\sim}{B}$

(c) State and prove : Lagrange's theorem.

(d) Define Reachable set . Also find node base of following graph.



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