Seat No.__________

Ganpat University

B.Tech. (CE/ IT) Sem - IV CBCS Regular Theory examination

Subject: 2HS401 Discrete Maths

May - June 2013

Time: - 3 Hours **Total Marks: 70**

Instructions:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION – I

Question-1 Attempt the following 12

(A) In $R-\{1\}$ there is a binary operation '$*$' defined $a*b=a+b-ab$, $\forall a,b\in R-\{1\}$. Is $\langle R-\{1\},*\rangle$ group?

(B) Define Group. Prove that $Z=\{0,1,2,3,4,5,6\}$ is commutative group under "addition modulo 7"

(C) Let $X=\left\{\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\Big/\alpha\in R\right\}$. Prove that $\langle X,\cdot\rangle$ is group with respect to usual matrix multiplication operation '.'

OR

Question-1 12

(A) Define sub group. Find all possible sub groups of group $G=\{0,2,4\}$ with respect to operation of addition modulo 4 (i.e. $+_4$)

(B) Define cyclic group. Show that $\langle Z,+\rangle$ is cyclic group. Find all possible generators of $\langle Z,+\rangle$

(C) Let G be a group. Prove the following results.

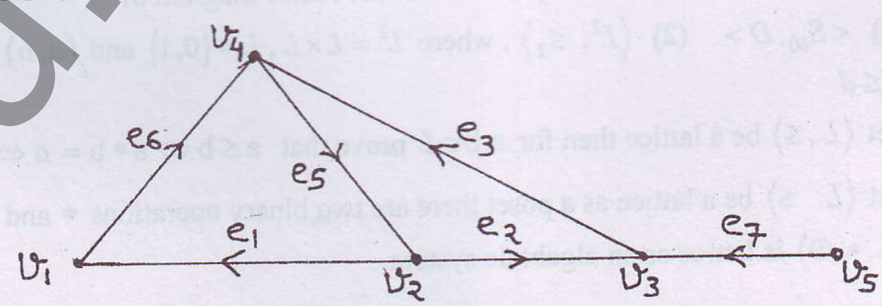
(1) If $a^{-1}=a \quad \forall a\in G$ then G is abelian group.

(2) Identity element in G is unique.

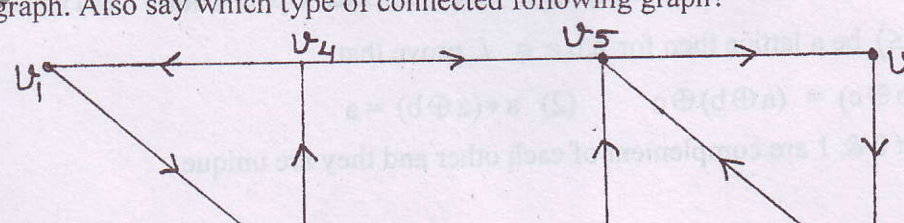
Question-2 Attempt the following 12

(A) Define: (1) Simple graph (2) Pseudo graph (3) Loop (4) Pendent vertex. Give example of each.

(B) Define Incident matrix representation of directed graph and find the Incident matrix of the following graph



(C) Define: (1) Weakly connected graph (2) Unilaterally connected graph (3) Strongly connected graph. Also say which type of connected following graph?



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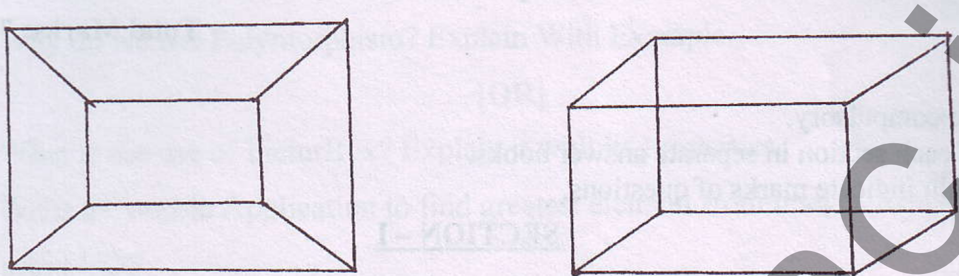
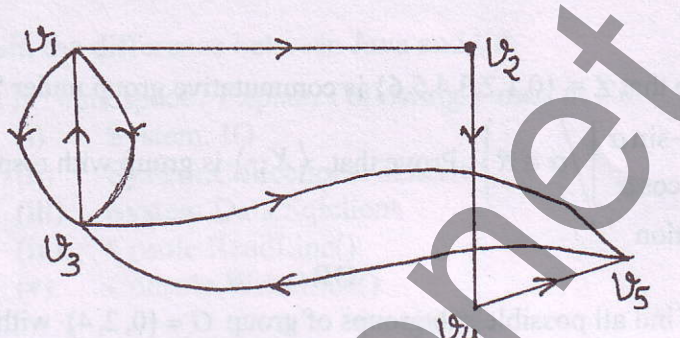
Question-2 **OR** **12**

(A) Define: (1) Cyclic graph (2) isolated vertex (3) Rich ability (4) Sub graph. Give example of each.

(B) Define Isomorphic graph. Also check whether the following graphs are isomorphic?



(C) Define path. Find all possible path form node v_1 to v_5 for following graph.



Question- 3 Attempt the following

(A) Define with example : (1) Fuzzy subset (2) Complement of Fuzzy subset **3**

(B) Let $G = \{1, -1, i, -i\}$. Show that $\langle G, \cdot \rangle$ is group with respect to usual multiplication operation ' . ' **4**

(C) If $A = \{(x_1 / 0.2), (x_2 / 0.7), (x_3 / 0.4), (x_4 / 1)\}$ $B = \{(x_1 / 0.1), (x_2 / 0.3), (x_3 / 0.2), (x_4 / 0.7)\}$ than find $A' + B, A \cdot B, A \cup B', B - A'$ **4**

SECTION – II

Question-4 Attempt the following **12**

(A) Explain Hasse Diagram of a poset. Draw the Hasse diagram of

(1) $< S_{60}, D >$ (2) $\langle L^2, \leq_2 \rangle$, where $L^2 = L \times L$, $L = \{0,1\}$ and $(a,b) \leq_2 (c,d)$ if $a \leq c$ and $b \leq d$

(B) Let $\langle L, \leq \rangle$ be a lattice then for a,b$\in L$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

(C) Let $\langle L \quad \leq \rangle$ be a lattice as a poset there are two binary operations $*$ and $\oplus$ on L such that $\langle L, *, \oplus \rangle$ is lattice as an algebraic system.

Question-4 **OR** **12**

(A) Define: (1) Lattice Isomorphism, (2) Poset (3) Least Upper bound (4) Chain

(B) Let $\langle L \quad \leq \rangle$ be a lattice then for a,b,c $\in L$ prove that

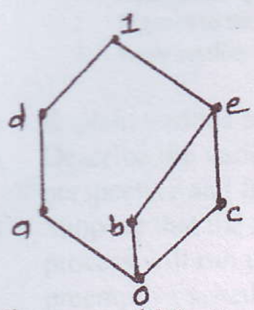
(1) $a \oplus (b \oplus c) = (a \oplus b) \oplus c$ (2) $a * (a \oplus b) = a$

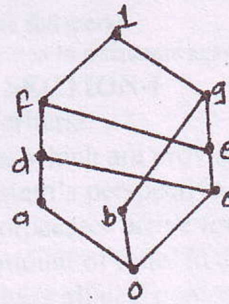
(C) Show that 0 & 1 are complement of each other and they are unique

Question-5 Attempt the following

(A) State Stone's representation theorem and explain it by giving an example. 3

(B) Define: Join irreducible element and Atom of a Boolean algebra. For the lattice in the following figures find the Join irreducible element and atoms. 4





(C) Define Minterm and Maxterm. Find all the minterms and maxterms of a Boolean algebra with two variables x_1 , x_2 . 4

Question-5 **OR**

(A) State & Prove De Morgan's theorem. 3

(B) B be a non-empty set , a be an element of $\langle B, *, \oplus, ', 0, 1\rangle$ Prove that $a \neq 0$, a is an atom iff $a * x = 0 \; or \; a * x = a, \forall x \in B$ where 0 means 0 – element 4

(C) Draw the Hasse diagram of $< S_{210}$, D > and find the join irreducible elements and atoms. 4

Question-6 Attempt the following (any three) 12

(A) Define poset. Show that $\langle P(X), \subseteq \rangle$ is a poset, where $P(X)$ is the power set of X

(B) Define complemented lattice and distributive lattice. Give an example of complemented lattice which is not a distributive lattice and explain it.

(C) Prove that every chain is a distributive Lattice.

(D) Define Boolean algebra and give any three examples of it with detail.

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