Seat No._____

GANPAT UNIVERSITY

B. TECH. SEM. IV CBCS (CE/IT)REGULAR EXAMINATION. April - June 2015

Sub : (2HS 401) DISCRETE MATHEMATICS

Total marks: 70

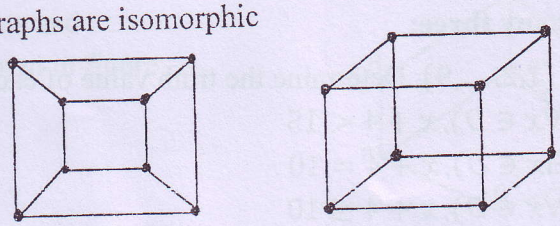
Time: 3 hrs

Instruction : (1) All questions are compulsory
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

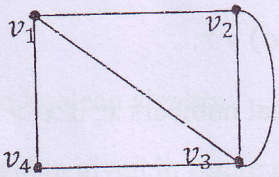
SECTION – I

Que-1 **Answer the following.** **(12)**

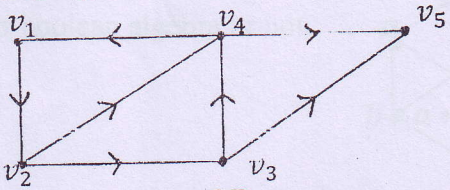
[A] Show that following graphs are isomorphic



[B] Write adjacency matrix representation of following graph.



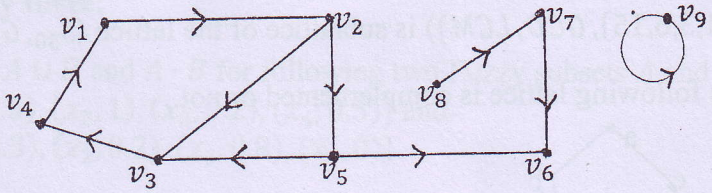
[C] Find reachable set of $\{v_1, v_3, v_5\}$ for the following graph.



OR

Que-1 **Answer the following.** **(12)**

[A] Find node base of following graph



[B] State and prove Hand shaking theorem.

[C] Consider the following directed graph $G(V, E)$ where
$V = \{1,2,3,4,5,6,7\}, E = \{(1,1), (2,5), (1,5), (5,2), (7,3), (1,5), (4,6), (4,2), (6,6)\}$

(1) Identify any loops.
(2) Identify any paralled edges.
(3) Are there any source in G?
(4) Are there any sinks in G?

Que-2 **Answer the following.**

[A] A subgroup H of a group G is normal if and only if $xHx^{-1} = H;\ \forall x \in G$. (04)

[B] Show that set of fourth root of unity form a group under multiplication. (04)

[C] Find all the subgroups of a cyclic group of order 20 with generator a. (03)

OR

Que-2 **Answer the following.**

[A] Show that the set of all positive rational numbers form an abelian group under the composition defined by $a * b = \frac{ab}{2}$. (04)

[B] Show that $(Z_4, +_4)$ is cyclic group and check $\bar{3}$ is generator of Z_4 or not. (04)

[C] If a^{-1} is the inverse of an element a of a group $(G,*)$ then $(a^{-1})^{-1} = a$. (03)

Que-3 **Attempt any three:** (12)

[A] Let $D = \{1,2,\ldots,9\}$. Determine the truth value of each of the following statement.

(1) $(\forall x \in D), x + 4 < 15$

(2) $(\exists x \in D), x + 4 = 10$

(3) $(\forall x \in D), x + 4 \leq 10$

(4) $(\exists x \in D), x + 4 > 15$

[B] Prove that $p \vee (q \vee r) = (p \vee q) \vee r$.

[C] Negate the statement, "for all real numbers x, if $x > 3$ then $x^2 > 9$".

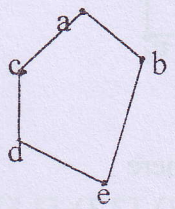
[D] Define group and graph.

SECTION –II

Que-4 **Answer the following.** (12)

[A] Check $\langle\{1,3,6,15\}, GCD, LCM\}\rangle$ is sublattice of the lattice $\langle S_{30}, GCD, LCM\rangle$ or not.

[B] Check the following lattice is complemented or not.



[C] Draw hasse diagram of $\langle S_{45}, D\rangle$ where aDb means a divides b.

OR

Que-4 Answer the following.

[A] Show that the partial order set $\langle N, D\rangle$ is lattice where N is set of natural number and aDb means a divides b.

[B] Let $\langle L, R\rangle$be a lattice and $b, c \in L$ then prove that $bRc \Rightarrow a\oplus b\ R\ a\oplus c, \forall a \in L$.

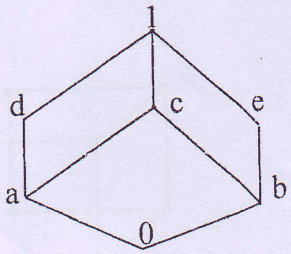
[C] Define (1) bounded lattice (2) complete lattice (3) complement of an element

Que-5 Answer the following.

[A] Write any six properties of boolean algebra. **(04)**

[B] Write the boolean expression $(x_1\oplus x_2)'\oplus(x_1' * x_3)$ in an equivalent sum of products canonical form in three variables x_1, x_2 and x_3. **(04)**

[C] Write join-irreducible elements, meet irreducible elements and atoms for the lattice shown in figure. **(03)**



OR

Que-5 Answer the following.

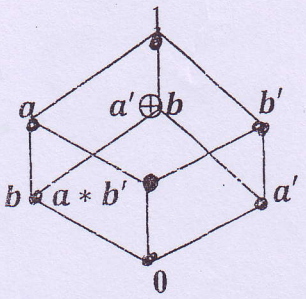
[A] Give an examples of two boolean algebra. **(04)**

[B] Consider the boolean algebra given in figure as under. Let the subsets of a given boolean algebra be as under. Check whether they are sub-boolean algebra or not. **(04)**

$S_1 = \{a, a', 0,1\}$

$S_2 = \{a * b', a'\oplus b, 0,1\}$

$S_3 = \{a * b', b', a, 1\}$



[C] Let A be a set, $A \neq \emptyset$. Then find join-irreducible elements and atoms for the lattice $\langle P(A), \subseteq\rangle$. **(03)**

Que-6 Attempt any three: **(12)**

[A] Find $A - B, A \cup B$ and $A \cdot B$ for following two Fuzzy subsets A and B where
$A = \{(x_1, 0.5), (x_2, 1), (x_3, 0.2), (x_4, 0.5)\}$ and
$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.8), (x_4, 0)\}$

[B] If A and B are fuzzy subsets of E then prove that $(A \cup B)' = A' \cap B'$.

[C] Define (1) Fuzzy subset (2) Boolean algebra.

[D] Verify $(A \cdot B)' = A' + B'$ for following two fuzzy subsets A and B.
$A = \{(x_1, 0.2), (x_2, 0.3), (x_3, 1), (x_4, 0)\}$ and
$B = \{(x_1, 0.3), (x_2, 0.5), (x_3, 0.7), (x_4, 1)$

END OF PAPER