Seat No.______

GANPAT UNIVERSITY

B.TECH.SEM.III (CIVIL) EXAMINATION. N0V/DEC - 2011

Sub : C301- Mathematics-III

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que-1 **(12)**

(a) $If\ L\{f(t)\} = \overline{f}\ (s)\ then\ Prove\ that\ L\left\{\int_0^t f(u)\,du\right\} = \frac{\overline{f}(s)}{s}$

(b) $Find\ (1)\ L\{e^{3t}\cos^2 t\}\quad (2)\ L^{-1}\left\{\frac{s+7}{s^2+2s+2}\right\}$

(c) $Find\ L^{-1}\left\{\log\left(\frac{s+7}{s-5}\right)\right\}$

OR

Que-1 **(12)**

(a) $If\ L\{f(t)\} = \overline{f}\ (s)\ then\ Prove\ that\ L\{\ t^n\ f\{t\}\} = (-1)^n\ \frac{d^n}{ds^n}\ \{\overline{f}\ (s)\}$

(b) $Find\ (1)\ L\ \{e^{-3t}u(t-2)\}\quad (2)\ L\left\{\frac{1-e^{-2t}}{t}\right\}$

(c) $State\ Convolution\ theorem\ and\ apply\ it\ to\ evaluate\ L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$

Que-2

(a) $Find\ a\ Fourier\ series\ for\ the\ function\quad f(x) = x - x^2\ ;\ [-\pi, \pi]$ **(03)**

$Hence\ show\ that\quad \frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots\ldots\ldots$

(b) $Obtain\ a\ Fourier\ series\ for\ the\ f^{ns}\ f(x)\ defined\ as$ **(04)**

$$f(x) = \begin{cases} \frac{\pi}{2} + x\ ; & -\pi \le x \le 0 \\ \frac{\pi}{2} - x\ ; & 0 \le x \le \pi \end{cases}$$

(c) $For\ -\pi < x < \pi\ \ Prove\ that\ ;$ **(04)**

$$sinax = \frac{2sina\pi}{\pi}\left[\frac{sinx}{1-a^2} - \frac{2sin2x}{2^2-a^2} + \frac{3sin3x}{3^2-a^2} - \dots\dots\dots\dots\right]$$

OR

Que-2

(a) $Expand\ f(x) = e^{2x}\ \ as\ a\ Fourier\ series\ \ for\ \ -1 < x < 1$ **(03)**

(b) $Find\ the\ fourier\ expansion\ of\ \ \ f(x) = x + x^2\ \ ;\ -1 \le x \le 1$ **(04)**

(c) $Obtain\ a\ Fourier\ series\ \ for\ the\ f^{ns}\ f(x)\ \ defined\ as$ **(04)**

$$f(x) = \begin{cases} 1 + x\ \ ;\ -1 \le x \le 0 \\ 1 - x\ \ \ ;\ \ \ 0 \le x \le 1 \end{cases}$$

Que-3 **(12)**

(a) $Use\ transform\ method\ to\ solve:\ y'' - 4y = 2\,e^{2t} + e^{4t}\ \ \ ;\ \ \ y(0) = 0$

(b) $Expand\ f(x) = \pi x - x^2\ \ \ as\ a\ Half-range\ \ sine\ series\ \ in\ \ \ 0 < x < \pi$

(c) $Find\ \ L^{-1}\left\{\frac{s^3}{s^4 - a^4}\right\}$

SECTION-II

Que-4 (12)

(a) Find an analytic function whose real part is $x^2 - y^2$

(b) Determine whether the function $w = f(z) = \sinh z$ is analytic or not.

(c) Find bilinear transformation which maps the points $z = 2, i, -2$ on to $w = 1, i, -1$.

OR

Que-4 (12)

(a) Evaluate $\int_0^{2+i} (\bar{z})^2 \, dz$, along the path $x = 2y$

(b) State and Prove Cauchy's theorem.

(c) Evaluate $\int_c \frac{z^2}{(z-1)} dz$, where C is the circles (i) $|z| = 1$ (ii) $|z| = \frac{1}{2}$

Que-5

(a) Form a partial differential equation by eliminating arbitrary constant or Function from (i) $z = ax + by + ab$ (ii) $z = f(x+it) + g(x-it)$ (04)

(b) Solve the Lagrange's equation: $(yz)p + (xz)q = xy$ (03)

(c) Using method of separation of variables solve the equation $r - 2p + q = 0$ (04)

OR

Que-5

(a) Form a partial differential equation by eliminating Function of $f\left[x^2 - y^2,\ x^2 - z^2\right] = 0$ (03)

(b) Solve: $x(y^2 - z^2) \cdot p + y(z^2 - x^2) \cdot q = z(x^2 - y^2)$ (04)

(c) Using method of separation of variables solve the equation (04)

$$2x\frac{\partial z}{\partial x} - 3y\frac{\partial z}{\partial y} = 0$$

Que-6 **Attempt any three** (12)

(a) Solve: $\frac{d^3 y}{dx^3} - 6\frac{d^2 y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{-2x} + e^{-3x}$

(b) Solve: (1) $(D^2 - 5D + 2)y = 0$ (2) $(D^3 + 1)y = 0$

(c) Solve: $\frac{d^2 y}{dx^2} + y = \sec x \tan x$; using variation of parameters method

(d) Solve using Cauchy's homogeneous method : $x^2\frac{d^2 y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$

END OF PAPER