Seat No.______

GANPAT UNIVERSITY
B. TECH. SEM. III (CIVIL) CBCS REGULAR EXAM. DEC. – 2013
Sub : (2CI 301) Mathematics – III

Time: 3 hrs **Total marks: 70**

Instruction : (1) All questions are compulsory
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Question-1 (12)

(A) Form a partial differential equation by eliminating arbitrary constant or Function from
(i) $z = (x-a)^2 + (y-b)^2$ (ii) $z = f(x^2 + y^2)$

(B) Solve : $(y^2 z)\,p + (x^2 z)\,q = x y^2$

(C) Solve : $x(y^2 - z^2)\,p + y(z^2 - x^2)\,q = z(x^2 - y^2)$

OR

Question-1 (12)

(A) Form a partial differential equation by eliminating arbitrary constant or Function from
(i) $\log(az - 1) = x + ay + b$ (ii) $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

(B) Solve : $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \left(\frac{x-y}{xy}\right)$

(C) Solve by the method of separation of variables : $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial u}{\partial x}$

Question-2

(A) Find the Complementary function of : $\frac{d^4y}{dx^4} + 4y = 0$ (03)

(B) Solve : $[D^3 - 6D^2 + 11D - 6]\,y = e^{-2x} + e^{-3x}$ (04)

(C) Solve Cauchy's homogeneous equation : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$ (04)

OR

Question-2

(A) Find the Particular Integral of : $[D^2 + 5D + 4]\,y = x^2 + 7x + 9$ (03)

(B) Solve : $[D^2 - 4D + 3]\,y = \cos 2x \sin 3x$ (04)

(C) Solve Legendre's equation : $(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 2\sin\{\log(1+x)\}$ (04)

Question-3 (12)

(A) Apply the method of variation of parameters to solve : $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

(B) Solve : $[D^3 - 3D^2 + 3D - 1]\,y = x^2 e^x$

(C) Solve : $\frac{\partial^2 u}{\partial x\,\partial t} = e^{-t}\cos x$; Where given that $u = 0$ when $t = 0$ AND $\frac{\partial u}{\partial t} = 0$ when $x = 0$

Section – II

Question-4 (12)

(A) Prove that : (1) $L\{\sin at\} = \frac{a}{s^2 + a^2}$ (2) $L\{e^{at}\} = \frac{1}{s-a}$

(B) Evaluate: (1) $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$ (2) $L\{t.\cos at\}$

(C) State and apply convolution theorem to evaluate: $L^{-1}\left\{\dfrac{1}{(S+1)(S^2+1)}\right\}$

Question-4 **OR** **(12)**

(A) Define Laplace transform and using definition, prove that $L\{t^n\} = \dfrac{\overline{n+1}}{s^{n+1}}$

(B) Using the definition find the Laplace transform $f(t) = \begin{cases} 2t & ; \quad 0 < t < 5 \\ 1 & ; \quad t > 5 \end{cases}$

(C) Using partial fraction ,find $L^{-1}\left\{\dfrac{2s-1}{(s+1)(s-2)}\right\}$

Question-5

(A) Find a Fourier series to represent $f(x) = x^3 \quad ; -\pi \le x \le \pi$ **(04)**

(B) Find a Fourier series to represent $f(x) = \begin{cases} -\pi & , \quad 0<x<\pi \\ x & \quad \pi<x<2\pi \end{cases}$ **(04)**

(C) Obtain Half range cosine series for $f(x) = 1 - x; \ 0 \le x \le 1$ **(03)**

Question-5 **OR**

(A) Show that Fourier series representation of $f(x) = x^2 \ ; -\pi \le x \le \pi$ is **(04)**

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$$

(B) Find a Fourier series to represent $f(x) = \begin{cases} a & , \quad 0<x<\pi \\ -a & \quad \pi<x<2\pi \end{cases}$ **(04)**

(C) Show that $0 \le x \le \pi$, the Fourier sine series representation of $f(x) = x(\pi - x)$ is **(03)**

$$f(x) = \frac{8}{\pi}\left(\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \ldots\ldots\ldots\right)$$

Question-6 : Attempt any three **(12)**

(A) Using Laplace transform method to solve: $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + 2y = 0$; Where

$y(0) = 1 \, , \ y'(0) = 0$

(B) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\left\{\int_0^t f(t)\,dt\right\} = \dfrac{\bar{f}(s)}{s}$

(C) State first shifting theorem and using it find $L\{e^{2t} \cos t \sin 3t\}$

(D) Find Half range sine series for $f(x) = e^x ; \ x \in (0, \pi)$

END OF PAPER