Seat No.______

# GANPAT UNIVERSITY

## B. TECH. SEM. III CBCS ( CIVIL ) EXAMINATION. NOV./DEC. – 2014

## Sub : (2HS 301) Mathematics – III

**Time: 3 hrs** **Total marks: 70**

**Instruction :** (1) All questions are compulsory
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.



## Section – I

**Que 1** (12)

(A) Solve : $[D^3 + 2D^2 + D]\, y = e^{2x} + x^2 + x$ .

(B) Solve : $[D^2 + 2D]\, y = 5\, e^{-2x} \sin x$ .

(C) Apply the method of variation of parameters to solve : $\frac{d^2y}{dx^2} + y = \sec x$

**OR**

**Que 1** (12)

(A) Solve : $[D^2 + 1]\, y = \sin 3x \cos 2x$ .

(B) Solve : $[D^2 + 2D]\, y = 5\, e^{-2x} \sin x$ .

(C) Apply the method of variation of parameters to solve : $\frac{d^2y}{dx^2} + 4y = \tan 2x$ .

**Que 2**

(A) Form a partial differential equation by eliminating arbitrary constant or function from (i) $z = ax + by + ab$ (ii) $z = f(x^2 - y^2)$ (04)

(B) Solve : $(y^2 z)\, p + (x^2 z)\, q = x y^2$ (04)

(C) Solve : $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$ by Direct integration . (03)

**OR**

**Que 2**

(A) Form a partial differential equation by eliminating arbitrary constants A & P from $z = A e^{Pt} \sin Px$ . (04)

(B) Solve : $(y^2)\, p - (xy)\, q = x(z - 2y)$ . (04)

(C) Solve : $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ by Direct integration . (03)

Que 3 **Attempt Any Two**

(A) Solve by the method of separation of variables : $2x\frac{\partial z}{\partial x} = 3y\frac{\partial z}{\partial y}$ .

(B) Solve Cauchy's homogeneous equation : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 12\,\log x$

(C) Solve the simultaneous equation : $\frac{dx}{dt} + y = \sin t\,, \frac{dy}{dt} + x = \cos t$ ;
Where given that $t = 0, x = 1$ & $y = 0$

## Section – II

Que 4 (12)

(A) If $L\{f(t)\} = \overline{f}(s)$ then Prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{\overline{f}(s)\}$

(B) Define Laplace transform and evaluate $L^{-1}\left\{\frac{s^3}{s^4 - a^4}\right\}$.

(C) Evaluate $L\{f(t)\}$ where $f(t) = \begin{cases} e^t \; ; \; t < 5 \\ 3 \; ; \; t \geq 5 \end{cases}$

OR

Que 4 (12)

(A) Using convolution theorem ; evaluate $L^{-1}\left\{\frac{4s-2}{(s^2+1)^2}\right\}$.

(B) Solve $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = -e^{-3t} + 3e^{-5t}$ ; $y(0) = 4, y'(0) = -14$ by Laplace transform method

(C) Define Laplace transform and evaluate $L^{-1}\left\{\frac{s+5}{s^2(s+2)}\right\}$.

Que 5

(A) Find the Fourier series of $f(x) = \begin{cases} \pi x \; ; & 0 \leq x \leq 1 \\ \pi(2-x) \; ; & 1 \leq x \leq 2 \end{cases}$ . (04)

(B) Obtain a Fourier cosine series for $f(x) = \begin{cases} \cos x \; ; & 0 < x < \frac{\pi}{2} \\ 0 \; ; & \frac{\pi}{2} < x < \pi \end{cases}$ (04)

(C) Obtain Fourier series for $f(x) = x \sin x$ in $-\pi \leq x \leq \pi$ (03)

OR

Que 5

(A) Expand $f(x) = \sqrt{1 - \cos x}$ as a Fourier series in the interval $0 \leq x \leq 2\pi$ and hence show that $\sum_{1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$ (04)

(B) Obtain a Fourier series for $f(x) = \begin{cases} -a \; ; & -l < x < 0 \\ a \; ; & 0 < x < l \end{cases}$ (04)

(C) Show that in $0 \leq x \leq \pi$ the half range Sine series for $x(\pi - x)$ is $\frac{8}{\pi}\left[\frac{\sin x}{1^2} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\cdots\right]$ (03)



2

e 6 **Attempt any three**

(A) Evaluate (1) $L\left\{\frac{1-e^{t}}{t}\right\}$ (2) $L\{t\cos(4t+3)\}$

(B) Evaluate : $L^{-1}\left\{\frac{3(1-e^{-\pi s})}{s^2+9}\right\}$

(C) Find Fourier series for $f(x) = e^{-x}$ ; $-c < x < c$.

(D) Prove that in the interval $0 < x < \pi$ ;

$$\frac{e^{ax}-e^{-ax}}{e^{a\pi}-e^{-a\pi}} = \frac{2}{\pi}\left[\frac{\sin x}{a^2+1} - \frac{2\sin 2x}{a^2+4} + \frac{3\sin 3x}{a^2+9} - \dots\dots\right]$$

***End Of Paper***