Seat No.:________

GANPAT UNIVERSITY

B. Tech. (CIVIL/CIVIL int.) Sem. III CBCS (NEW) Examination , NOV. – 2015

Sub : (2HS 305) Mathematics For Civil Engineering

Time: 3 hrs | Total marks : 60

Instruction : (1) All questions are compulsory
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que 1

(A) If $L\{f(t)\} = \bar{f}(s)$ then Prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{\bar{f}(s)\}$ (03)

(B) Using convolution theorem ; evaluate $L^{-1}\left\{\frac{1}{(s-3)(s+7)}\right\}$. (03)

(C) Evaluate : (i) $L\left\{\frac{\cos^2 t}{t}\right\}$ (ii) $L\{t\, e^{-2t} \sin 4t\}$ (04)

OR

Que 1

(A) Evaluate : $L^{-1}\left\{\log\left(\frac{s+4}{s-6}\right)\right\}$ (03)

(B) Using Definition find Laplace transforms : $f(t) = \begin{cases} 0\ ; & 0 < t < 3 \\ 1\ ; & 3 < t < 5 \\ 0\ ; & t \geq 5 \end{cases}$ (03)

(C) Evaluate : $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$ (04)

Que 2

(A) Expand $f(x) = 1 - x^2$ as a fourier series in the range $-1 \leq x \leq 1$. (03)

(B) Obtain Fourier series for $f(x) = \begin{cases} \frac{\pi}{2} + x\ ; & -\pi < x < 0 \\ \frac{\pi}{2} - x\ ; & 0 < x < \pi \end{cases}$. (03)

(C) Obtain Fourier series for $f(x) = e^{-x}$; $0 < x < 2\pi$. (04)

OR

Que 2

(A) Prove that for $-\pi < x < \pi$; (03)

$$\sin ax = \frac{2 \sin a\pi}{\pi}\left[\frac{\sin x}{1-a^2} - \frac{2\sin 2x}{2^2-a^2} + \frac{3 \sin 3x}{3^2-a^2} - \dots\dots\right]$$

(B) Obtain Half range Cosine series for $f(x) = x^2$ in the range $0 \leq x \leq \pi$. (03)

(C) Obtain a Fourier series for $f(x) = \begin{cases} 1+x\ ; & -1 < x < 0 \\ 1-x\ ; & 0 < x < 1 \end{cases}$ (04)

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots$

Que: 3 **Attempt Any Two** (10)

(i) Solve $\frac{dy}{dt} + 2y = 10\,e^{3t}$; $y(0) = 6$ by Laplace transform method .

(ii) Define Unit step Function . Express the given function in terms of unit step function and hence obtain its Laplace transforms : $f(t) = \begin{cases} \sin t \; ; & t < \pi \\ t \; ; & t \geq \pi \end{cases}$

(iii) Obtain Fourier series for $f(x) = x + x^2$; $-\pi \leq x \leq \pi$.

Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots$

Section – II

Que 4

(A) Solve : $[D^2 - 6D + 9]\,y = e^{3x}$. (03)

(B) Solve : $[D^3 + 2D^2 + D]\,y = x^2 + x$. (03)

(C) Apply the method of Variation of Parameters to solve : $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. (04)

OR

Que 4

(A) Solve : $[D^2 + 9]\,y = \cos 3x$. (03)

(B) Solve : $[D^2 - 2D + 2]\,y = e^x \sin x$. (03)

(C) Apply the method of Variation of Parameters to solve : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ (04)

Que 5

(A) Form a partial differential equation by eliminating arbitrary constants or function from (i) $z = (x^2 + a)(y^2 + b)$ (05)

(ii) $f(lx + my + nz \,, x^2 + y^2 + z^2) = 0$

(B) Solve : $(y^2 z)\,p + (x^2 z)\,q = x y^2$ (05)

OR

Que 5

(A) Solve : $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$; for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. (05)

(B) Solve : $x(y^2 - z^2)\,p + y(z^2 - x^2)\,q = z(x^2 - y^2)$ (05)

Que 6 (10)

Attempt any two

(i) Solve Cauchy's Homogeneous equation : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$

(ii) Solve the Simultaneous equation : $\frac{dx}{dt} = 7x - y$, $\frac{dy}{dt} = 2x + 5y$.

(iii) Solve by the method of Separation of variables : $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$.

END OF PAPER