Seat No. ________

GANPAT UNIVERSITY

B.TECH. (EC) SEM. – III REGULAR THEORY

SUBJECT: 2HS301 ENGINEERING MATHEMATICS – III

NOVEMBER – DECEMBER 2011

TIME: - 3 HOURS **TOTAL**

MARKS: 70

INSTRUCTIONS:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION – I

Que-1 **(12)**

[A] Derive $L\{\sin at\}$ and find $L^{-1}\left\{\dfrac{1}{s\left(s^2+a^2\right)}\right\}$

[B] Find: (1) $L\left\{e^{3t}\sin 2t\cos 3t\right\}$ (2) $L^{-1}\left\{\log\left(1-\dfrac{1}{S^2}\right)\right\}$

[C] Use transform method to solve:

$$y''-3y''-y'-2y=4t+e^{3t}\;;\quad y(0)=1,\; y'(0)=-1.$$

OR

Que-1 **(12)**

[A] If $L\{f(t)\}=\bar{f}(S)$ then Prove that $L\left\{t^n f(t)\right\}=(-1)^n\dfrac{d^n}{dS^n}\left[\bar{f}(S)\right]$; where n = 1,2,3…

[B] Find: (1) $L\left\{\dfrac{\sin at}{t}\right\}$ (2) $L^{-1}\left\{\dfrac{S}{S^4+4}\right\}$

[C] Define unit step function. Express given function in terms of the unit step function Hence obtain its Laplace transform. $f(t)=\begin{cases} S\text{int} & ; \; t<\pi \\ t & ; \; t\geq\pi \end{cases}$

Que-2

[A] Find a Fourier series for the function: $f(x) = x^2 - 2\,,\ -2 \le x \le 2$ **(03)**

[B] Find a Fourier series to represent $f(x) = x - x^2\,,\ -\pi \le x \le \pi$ **(04)**

Hence show that $\dfrac{\pi^2}{12} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + - -$

[C] Find the half range sine series for $f(x) = \begin{cases} \dfrac{\pi x}{4} & ; \ 0 \le x \le \dfrac{\pi}{2} \\ \dfrac{\pi(\pi - x)}{4} & ; \ \dfrac{\pi}{2} \le x \le \pi \end{cases}$ **(04)**

OR

Que-2

[A] Find a Fourier series to represent: $f(x) = x^2\,,\ -\pi \le x \le \pi$ **(03)**

Hence deduce that : $\dfrac{\pi^2}{6} = \dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \ldots\ldots$

[B] Find a Fourier series for the function : $f(x) = \begin{cases} -x & ; \ -\pi \le x \le 0 \\ x & ; \ 0 \le x \le \pi \end{cases}$ **(04)**

Hence deduce that : $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots\ldots$

[C] Find the half range sine series to represent $f(x) = x \sin x\ ,\ 0 < x < \pi$ **(04)**

Que-3 **Attempt any three:** **(12)**

[A] State and Prove convolution theorem

[B] Find: (1) $L\{\, t\, e^{2t} \cos 3t \}$ (2) $L\{\sinh 4t \sinh 2t\}$

[C] Find a Fourier series to represent the function $f(x) = x \sin x,\ -\pi \le x \le \pi$

[D] Find the half range sine series for $f(x) = \begin{cases} x & ; \ 0 \le x \le \dfrac{\pi}{2} \\ \pi - x & ; \ \dfrac{\pi}{2} \le x \le \pi \end{cases}$

SECTION – II

Que-4 **(12)**

[A] Find the root of $f(x) = x^3 - x - 1$, using Newton Raphson method correct up to three decimal places, taking initial point a = 3.

[B] Find the root of $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$, $y(1) = 2$, using Euler's method at x = 1.5 in five steps.

[C] Solve the following system of linear equations using Gauss – Seidel method

27x + 6 y – z = 85, 6x + 5y + 2z = 72, x + y + 54z = 110

OR

Que-4 **(12)**

[A] Find the root of equation $x^3 - 3x + 4 = 0$ by using False Position method correct up to three

decimal places.

[B] Obtain Picard's second approximate solution of the initial value problem

$\frac{dy}{dx} = x^2 + y^2$, $y(0) = 0$ For x = 0.4 correct to four decimal places.

[C] Solve the following system of linear equations using Gauss – Jorden method

2x + y +4z = 12, 8x - 3y + 2z = 20, 4x + 11y - z = 33.

Que-5

[A] State & Prove Cauchy's theorem. **(03)**

[B] If $f(z) = u + iv$ is an analytic function of z then find $f(z)$ where **(04)**

$u - v = e^x(Cosy - Siny)$

[C] Evaluate : $\int_c \frac{dz}{z^2 - 2z}$ where c is the circle $|z - 2| = 1$ **(04)**

OR

Que-5

[A] Find the Bilinear transformation which maps the points $z = 2, i, -2$ in to the points $w = 1, i, -1$ **(03)**

[B] Find an analytic function whose imaginary part is $(x^2 - y^2) + \frac{x}{x^2 + y^2}$ **(04)**

[C] Evaluate : $\int_c \frac{z^2+z+1}{z^2-7z+2}\,dz$ where c is the ellipse $4x^2+9y^2=1$ **(04)**

Que-6 **<u>Attempt any three:</u>** **(12)**

[A] Evaluate: $\int_0^{\frac{\pi}{2}} \sin x\, dx$ by Simpson's one third rule using 11 ordinates.

[B] Find f'(2) & f''(2) from the following observation table

x	1.5	2.0	2.5	3.o	3.5	4.0
y	3.375	7.0	13.625	24.0	38.875	59.0

[C] Following table gives the population of town. Find the increases in population during 1946 to 1948.

X	1911	1921	1931	1941	1951	1961
Y	12	15	20	27	39	52

[D] Solve the difference equation : $y_{n+2}-7y_{n+1}+10y_n=4^n+12e^{3n}$

END OF PAPER