Student Exam No._____

GANPAT UNIVERSITY

B. Tech. Semester: III (EC)

Regular Examination November – December 2013

2HS301 – ENGINEERING MATHEMATICS - III – Theory

Time: 3 Hours **Total Marks:** 70

Instruction: 1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION –I

Que.-1 **Attempt the following:** **12**

(A) State & Prove Cauchy's theorem.

(B) If $f(z)=u+iv$ is an analytic function of z then find $f(z)$ where $u-v=e^{x}\left(Cosy-Siny\right)$

(C) Evaluate: $\int_{0}^{2+i}\left(\overline{Z}\right)^{2}dz$; along the real axis to 2 and the vertically to $2+i$.

OR

Que.-1 **Attempt the following:** **12**

(A) Find the Bilinear transformation which maps the points $z=1,i,-1$ in to the points $w=i,0,-i$ respectively.

(B) If $f(z)$ is an analytic function of z then prove that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\left|f(z)\right|^2=4\left|f'(z)\right|^2$.

(C) Evaluate : $\int_{c}\frac{e^{2z}}{\left(z+1\right)^4}dz$; where c is the circle $|z|=2$.

Que.-2 **Attempt the following:** **11**

(A) Prove that $\Delta\log f(x)=\log\left[1+\frac{\Delta f(x)}{f(x)}\right]$; the interval of difference being unity. **3**

(B) In a certain experiment; the values of x and y were found as under. Find the value of y when x=2.5. **4**

X	0	1	2	3	4	5	6
y	0	1	16	81	256	625	1296

(C) Find the value of y when x = 9 from the following data **4**

x	5	7	11	13	17
y	150	392	1452	2366	5202

OR

Que.-2 **Attempt the following:** **11**

(A) Show that $\Delta^2\left(\cos 2x\right)=-4\sin^2 h\cos\left(2x+2h\right)$. **3**

(B) The populations of a country in the decennial census were as under. Estimate the population for the year 1975. **4**

Year (x)	1941	1951	1961	1971	1981
Population (y)	46	67	83	95	102

(C) The following table gives the barometric pressure P at height H above the sea level. Find P when H = 5000. **4**

X	20	30	40	50
N	630	525	450	270

Que.-3 Attempt any three: 12

(A) Find the Fourier series for the function $f(x)=x+x^2$; $[-\pi,\pi]$

Hence show that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+---$

(B) Obtain Fourier series for the function $f(x)$ given by

$$f(x)=\begin{cases}1+\frac{2x}{\pi} & ; \quad -\pi\le x\le 0\\ 1-\frac{2x}{\pi} & ; \quad 0\le x\le\pi\end{cases}$$

(C) Find a series of sine of multiples of x which represent $f(x)$ in $(0,\pi)$ where

$$f(x)=\begin{cases}\frac{\pi x}{4} & ; \quad 0\le x\le\frac{\pi}{2}\\ \frac{\pi}{4}(\pi-x) & ; \quad \frac{\pi}{2}\le x\le\pi\end{cases}$$

(D) Obtain the half range cosine series for $f(x)=x^2$; $[-\pi,\pi]$

SECTION –II

Que.-4 Attempt the following: 12

(A) If $L\{f(t)\}=\bar{f}(s)$ then prove that $L\{t^n f(t)\}=(-1)^n\frac{d^n}{ds^n}\bar{f}(s), n=1,2,3,\ldots$

(B) State convolution theorem and use it to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$.

(C) Find $L\{f(t)\}$ where $f(t)=\begin{cases}\sin t, t<\pi\\ t, t\ge\pi\end{cases}$

OR

Que.-4 Attempt the following: 12

(A) Solve $y''+9y=r(t)$ by Laplace transform method where $y(0)=0, y'(0)=4$, $r(t)=8sint$ if $0<t<\pi$ and 0 if $t>\pi$

(B) Find the Laplace transform of the Saw-tooth wave function $f(t)=\frac{k}{p}t$, if $0<t<p$, $f(t+p)=f(t), \forall t$

(C) Evaluate (1) $L\left\{\frac{e^{-t}\sin t}{t}\right\}$ (2) $L^{-1}\left\{log\left(1+\frac{4}{s^2}\right)\right\}$

Que.-5 Attempt the following: 11

(A) Find a real root of the equation $x\sin x+\cos x=0$ using Newton Raphson method correct to three decimal places. 4

(B) Using Taylor's series method obtain the solution of $\frac{dy}{dx}-x^2=y^2$ where $y(0)=1$. Also find the values of y at $x=0.1$. 4

(C) Evaluate $\int_0^1\frac{dx}{1+x^2}$ using Simpson's $3/8^{th}$ rule taking three equal parts. Hence obtain an approximate value of π. 3

OR

Que.-5 Attempt the following: 11

(A) Find positive root of the equation $x^4 - x - 10 = 0$ by self iteration method up to four decimal places. 4

(B) Apply Euler's method to solve $\frac{dy}{dx} = x + y^2$ for $x = 0.5$ given that $y = 1$ when $x = 0$ and $h = 0.1$. 4

(C) Use Simpson's $1/3^{rd}$ rule to find $\int_0^{0.6} e^{-x^2}\,dx$ by taking seven ordinates. 3

Que.-6 Attempt any three: 12

(A) Evaluate $L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{5/2}}\right\}$

(B) Solve $y''' + 2y'' - y' - 2y = 0$ by Laplace transform method where $y(0) = 1$, $y'(0) = 2, y''(0) = 2$

(C) Solve the following system of equation by Gauss Seidal method
$30x - 2y + 3z = 75,\ x + 17y - 2z = 48,\ 2x + 2y + 18z = 30$ correct up to four decimal places.

(D) Find the first and second ordered derivatives of the function tabulated below, at the point $x = 2.2$

x	1.2	1.4	1.6	1.8	2	2.2
$y = f(x)$	3.32	4.055	4.95	6.055	7.39	9.025

END OF PAPER