GANPAT UNIVERSITY

B. Tech. Semester VI (Electronics and Communication Engineering)

CBCS Regular Theory Examination May – June 2013

(2EC604) Introduction to Detection Theory

Time: 3 Hours **Total Marks: 70**

Instruction:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.
4. Standard terms and notation are used.

Section - I

Q – 1 **(A)** A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. There are total 16 articles. Two articles are chosen from the lot at random (without replacement). Find the probability that (I) both are good (II) both have major defects (III) at least 1 is good (IV) at most 1 is good (V) exactly 1 is good (VI) neither is good. [6]

(B) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y). [6]

OR

Q – 1 **(A)** A random variable X has the following probability distribution, [6]

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find (I) the value of K (II) $P(1.5 < X < 4.5/X > 2)$ (III) the smallest value of λ for which $p(X \le \lambda) > 1/2$

(B) For probability distribution of (X,Y) given below, Find: $P(X \le 1)$, $P(Y \le 3)$, $P(X \le 1, Y \le 3), P(X \le 1/Y \le 3), P(Y \le 3/X \le 1)$ *and* $P(X+Y \le 4)$. [6]

X	Y: 1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Q – 2 Find out mean and variance of following distributions. Probability density functions are given by

(A) Binomial distribution : $P(X=r) = nCr\, p^r q^{n-r}$ *where* $r = 0,1,\ldots,n,\ p+q=1$ [4]

(B) Gaussian distribution : $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ *where* $-\infty < x < \infty, -\infty < \mu < \infty$ *and* $\sigma > 0$ [4]

(C) Exponential distribution : $f(x) = \lambda e^{-\lambda x}$ *where* $x \ge 0, \lambda > 0$ [3]

OR

Q – 2 Find out mean and variance of following distributions. Probability density functions are given by

(A) Poisson distribution : $P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!}$ *where* $r = 0,1,2 \ldots \infty$ *and* $\lambda > 0$ [4]

(B) Rayleigh distribution : $f(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}}$ *where* $x \ge 0$ *and* $\sigma > 0$ [4]

(C) Uniform distribution : $f(x) = \frac{1}{b-a}$ *where* $a < x < b$ [3]

Q – 3 **(A)** The cdf of a continuous RV X is given by [6]

(I) Find pdf of X.

(II) evaluate $P(|X| \le 1)$ and $P(\frac{1}{3} \le X < 4)$ using both pdf and cdf.

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < \frac{1}{2} \\ 1 - \frac{3}{25}(3-x)^2 & \frac{1}{2} \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

(B) Given $f_{xy}(x,y) = xy^2 + \frac{x^2}{8}$, *where* $0 \le x \le 2$ *and* $0 \le y \le 1$. [6]

Compute $P(X > 1)$, $P\left(Y < \frac{1}{2}\right), P\left(X > 1/Y < \frac{1}{2}\right)$ *and* $P(Y < \frac{1}{2}/X > 1)$.

Section – II

Q – 4 **(A)** X and Y are random variable and if $U = \frac{X-a}{h}$ and $V = \frac{Y-b}{k}$ *where* $h, k > 0$ then prove that coefficient of correlation $r_{XY} = r_{UV}$ **[4]**

(B) If X denotes the sum of the numbers obtained when 2 dice are thrown, obtain an upper bound for $P\{|X - 7| \geq 4\}$. Compare with exact probability. **[4]**

(C) If $X(t)$ is a wide sense stationary process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine $E\left[\left(X(8) - X(5)\right)^2\right]$ **[4]**

OR

Q – 4 **(A)** Compute the coefficient of correlation between X and Y using following data: **[4]**

X	1	3	5	7	8	10
Y	8	12	15	17	18	20

(B) If X is a RV with $E[X] = \mu$ and $VAR[X] = \sigma^2$, then Prove that **[4]**

$P\{|X - \mu| \geq c\} \leq \frac{\sigma^2}{c^2}$ where $c > 0$.

(C) Find out the value of autocorrelation of output process if any random process passes through the linear system. **[4]**

Q – 5 **(A)** Explain the probability of error for the detection of binary signals. **[6]**

(B) Derive the equation of transfer function of optimum filter. **[5]**

OR

Q – 5 **(A)** Compare M-ary PSK and M-ary FSK. **[6]**

(B) (I) Prove bayes' rule. **[5]**

(II) Prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(C \cap B) + P(A \cap B \cap C)$$

Q – 6 **(A)** A dart is thrown nine times at a target consisting of three areas. Each throw has a probability of 0.2, 0.3 and 0.5 of landing in areas 1, 2 and 3 respectively. Find the probability that the dart land exactly three times in each of the areas. **[6]**

(B) Explain binary Maximum Likelihood (ML) detection. **[6]**

END OF PAPER