GANPAT UNIVERSITY

B. Tech. Semester VI (Electronics and Communication Engineering)
CBCS Regular Theory Examination May – June 2014
(2EC604) Introduction to Detection Theory

Time: 3 Hours **Total Marks: 70**

Instruction:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.
4. Standard terms and notation are used.
5. Assume suitable data if necessary.

Section - I

Q – 1 (A) If $P(x) = xe^{-x^2/2},\ x \geq 0$ **[06]**

(I) Show that P(x) is a pdf of continuous RV X.
(II) Find its cumulative distribution function F(x).

(B) Find the Coefficient of correlation between X and Y using following data: **[06]**

X	65	67	66	71	67	70	68	69
Y	67	68	68	7	64	67	72	70

OR

Q – 1 (A) If the density function of a continuous RV X is given by $f(x)$. (I) Find value of a (II) find cdf of X (III) find $P(X > 1.5)$ **[06]**

$$f(x) = \begin{cases} ax & 0 \leq x \leq 1 \\ a & 1 \leq x \leq 2 \\ 3a - ax & 2 \leq x \leq 3 \\ 0 & elsewhere \end{cases}$$

(B) The joint density function of the RVs X and Y is given by $f(x, y) = 8xy\ \ where\ 0 < x < 1, 0 < y < x.$ Find $P\left(Y < \frac{1}{8} / X < \frac{1}{2}\right)$ **[06]**

Q – 2 (A) Find out mean and variance of Gaussian distribution : **[06]**

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad where -\infty < x < \infty, -\infty < \mu < \infty\ and\ \sigma > 0$$

(B) If X is a RV with $E[X] = \mu$ and $VAR[X] = \sigma^2$, then Prove that $P\{|X - \mu| \geq c\} \leq \frac{\sigma^2}{c^2}$ where $c > 0$. **[05]**

OR

Q – 2 (A) Find out mean and variance of Binomial distribution : **[06]**

$$P(X = r) = nCr\, p^r q^{n-r}\ \ where\ r = 0,1, \ldots, n,\ \ p + q = 1$$

(B) If the RV X is uniformly distributed over $(-\sqrt{3}, \sqrt{3})$, compute $P\left\{|X - \mu| \geq \frac{3\sigma}{2}\right\}$ and compare it with the upper bound obtained by Tchebycheff's inequality. **[05]**

Q – 3 (A) A random variable X has the following probability distribution, **[06]**

x	1	2	3	4	5	6	7
$P(x)$	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find (I) the value of K (II) $P(1.5 < X < 4.5 / X > 2)$ (III) the smallest value of λ for which $p(X \leq \lambda) > 1/2$

(B) Find out mean and variance of following distributions. Probability density functions are given by **[06]**

1. Uniform distribution : $f(x) = \frac{1}{b-a}$ $\quad where\ a < x < b$
2. Exponential distribution : $f(x) = \lambda e^{-\lambda x}$ $\quad where\ x \geq 0, \lambda > 0$

Section – II

Q – 4 **(A)** One integer is chosen at random from the number 1, 2, 3,…, 100.what is the probability that the chosen number is divided by (i) 6 or 8 and (ii) 6 or 8 or both? **[06]**

(B) An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball **[06]**

OR

Q – 4 **(A)** Answer the following questions.

1) If $A \subset B$ and $A \cap B = A$, what is the answer of P (B/A)? **[02]**
2) What is the $P(\bar{A} \cap \bar{B})$ if A and B are the independent events? **[02]**
3) If P (A) > P (B), P (A/B) is grater or less than P (B/A)? **[02]**

(B) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (i) a '1' is received and (ii) a '1' was transmitted given that a '1' was received. **[06]**

Q – 5 **(A)** State and explain the properties of matched filter. **[06]**

(B) Find the variance of the stationary process {X(t)}, whose auto-correlation function (ACF) is given by $R(\tau) = 36 + \frac{50}{1+6\tau^2}$ **[05]**

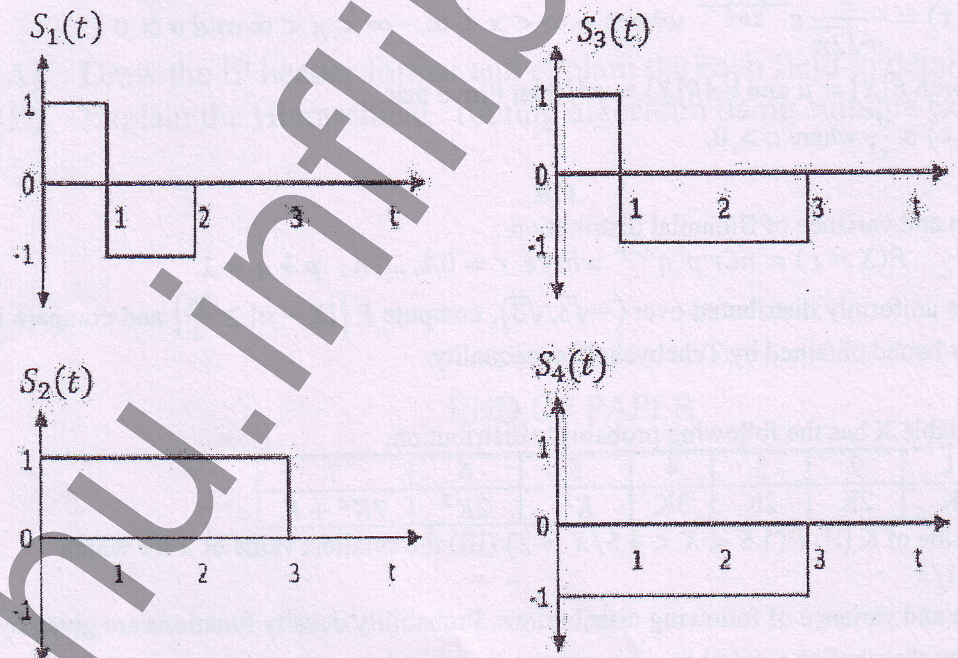
OR

Q – 5 **(A)** Explain optimum detector. **[06]**

(B) If X(t) is a wide sense stationary process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine $E\left[\left(X(10) - X(4)\right)^2\right]$ **[05]**

Q – 6 **(A)** Describe the difference between wide-sense stationary process and strict-sense stationary process. **[06]**

(B) Find the orthogonal basis signals for following signals using Gram-Schmidt procedure. **[06]**



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