Seat No. ______

# GANPAT UNIVERSITY

## B.Tech. ( E.E ) Sem-III CBCS Regular Theory Examination.

## SUBJECT: 2HS 301 Engineering Mathematics – III

## Nov-Dec 2012.

**TIME: - 3 HOURS** **TOTAL MARKS: 70**

**INSTRUCTIONS:**

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

## SECTION – I

**Question-1 Attempt the following:** (12)

(A) Evaluate : (1) $L\left\{e^{t}(2\cos 3t+4\sin 5t)\right\}$ (2) $L\left\{\dfrac{e^{-at}-e^{-bt}}{t}\right\}$

(B) Evaluate: (1) $L^{-1}\left\{\dfrac{1}{(S-2)^{2}+4}\right\}$ (2) $L^{-1}\left\{\dfrac{S}{(S^{2}+1)(S^{2}+9)}\right\}$

(C) Using Laplace method, solve the initial value problem $\dfrac{d^{2}y}{dt^{2}}+5\dfrac{dy}{dt}+6y=e^{t}$, where $y(0)=0$ , $y'(0)=1$

**Question-1**

OR (12)

(A) Find the Laplace Transform of $f(t)=\begin{cases}(t-1)^{2}, & 0<t<1\\ 0, & t\geq 1\end{cases}$

(B) If $L\{f(t)\}=\overline{f(s)}$ , prove that $L\{t^{n}.f(t)\}=(-1)^{n}\dfrac{d^{n}}{ds^{n}}L\{f(t)\}$. Using it find $L\{t.\cosh t\}$

(C) State convolution theorem and apply it to evaluate: $L^{-1}\left\{\dfrac{1}{(S+1)(S^{2}+1)}\right\}$

**Question-2 Attempt the following:**

(A) Expand : $f(x)=e^{x}$ in a half range cosine series in the interval $[0,\ 1]$ (03)

(B) Find a Fourier series for the function : $f(x)=\begin{cases}-x^{2} & ; \quad 0\leq x\leq \pi\\ x^{2} & ; \quad \pi\leq x\leq 2\pi\end{cases}$ (04)

(C) Find a Fourier series representation of $f(x)=x+x^{2}$ , $-\pi<x<\pi$ (04)

**Question-2**

OR

(A) Find the half range cosine series to represent $f(x)=1-x,\ 0\leq x\leq 1$ (03)

(B) Find a Fourier series to represent the function $f(x)=\cosh x$ , $-\pi\leq x\leq \pi$ (04)

(C) Find a Fourier series for the function: $f(x)=\sqrt{1+\cos x}$ , $-\pi<x<\pi$ (04)

**Question-3 Attempt any three:** **(12)**

**(A)** Find the Fourier integral of $f(x) = \begin{cases} 1 & ; \ |x| \le 1 \\ 0 & ; \ |x| > 1 \end{cases}$ and hence evaluate

(1) $\int_0^{\infty} \frac{\sin\lambda . \cos \lambda x}{\lambda} \, d\lambda$ and (2) $\int_0^{\infty} \frac{\sin\lambda .}{\lambda} \, d\lambda$

**(B)** Find Fourier cosine transform of $e^{-2|x|}$. Hence show that $\int_0^{\infty} \frac{\cos\lambda x}{\lambda^2 + 4} \, dx = \frac{\pi}{2} e^{-2|x|}$

**(C)** Using partial fraction method, find $L^{-1}\left\{\frac{1}{(S+2)^2(S-2)}\right\}$

**(D)** (1) Find : $L\{cost.cos2t.cos3t\}$ (2) Prove that: $L\{e^{at}\} = \frac{1}{s-a}$ ; $s > a$

**SECTION – II**

**Question-4 Attempt the following:** **(12)**

**(A)** Solve: (1) $\frac{d^4x}{dt^4} = m^4 x$ (2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

**(B)** By variation of parameter method solve: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} - 9y = \frac{e^{3x}}{x^2}$

**(C)** Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = 0$ given that $x = 0, y = 0$ and $\frac{dy}{dx} = 1$

**OR**

**Question-4** **(12)**

**(A)** Solve the simultaneous equation : $\frac{dx}{dt} + 7x - y = 0$ and $\frac{dy}{dt} + 2x + 5y = 0$

**(B)** Solve: $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12 \log x}{x^2}$

**(C)** Solve: (1) $(D^2 + D)y = x^2 + 2x + 4$ (2) $(D^2 - 2D + 1)y = e^x$

**Question-5 Attempt the following:** **(11)**

**(A)** Verify Cayley – Hamilton theorem & *find* $A^{-1}$ for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

**(B)** For two independent events A and B if $P(A) = 0.3$ and $P(A \cup B) = 0.6$ then find $P(B)$

**(C)** If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$ show that $A^* A$ is Hermitian matrix.

**OR**

**Question-5** **(11)**

(A) Define Hermitian matrix & $skew - Hermitian\ matrix\ by\ giving\ an\ example.$

(B) If $P(A) = \frac{1}{3}, P(B') = \frac{1}{4} \& P(A \cap B) = \frac{1}{6}$ the find $P(A \cup B)$ & $P(A' \cap B')$

(C) Diagonalise the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

**Question-6** **Attempt any three** **(12)**

(A) Define an analytic function & $show\ that\ w = coshz\ is\ analytic\ \ find\ its\ derivative.$

(B) State and prove cauchy's theorem incomplex theory.

(C) State cauchy's integral formula & $find \oint \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ where $C: |z| = 3$

(D) Find the fixed points & $normal\ form\ for\ given\ bilinear\ transformation$

$$w = \frac{5z + 4}{z + 5}$$

**END OF PAPER**