Seat No. ______

GANPAT UNIVERSITY

B. TECH. SEM. III (EE) EXAMINATION. NOV/DEC – 2014

Sub: (2HS301) Engineering Mathematics – III

Time: 03 hrs. Total marks: 70

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Que-1 (12)

(A) If $L\{f(t)\} = \overline{f}(s)$ then Prove that $L\left\{\int_0^t f(u)\,du\right\} = \frac{\overline{f}(s)}{s}$

(B) Find : (1) $L\{e^{-t}\cos^2 4t\}$ (2) $L\{t\,e^{-t}\sin 5t\}$

(C) State Convolution theorem and apply it to evaluate $L^{-1}\left\{\frac{1}{(s-2)(s+3)}\right\}$

OR

Que-1 (12)

(A) Evaluate : (1) $L\left\{\frac{1-e^t}{t}\right\}$ (2) $L^{-1}\left\{\frac{s^2+6}{(s^2+1)(s^2+4)}\right\}$

(B) Solve : $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$; where $y(0) = 0$; $y'(0) = 1$.

(C) Express the given function in terms of the unit step function and hence obtain it's laplace transforms : $f(t) = \begin{cases} -1; & t < 4 \\ 1; & t \geq 4 \end{cases}$

Que-2

(A) Find a Fourier series for the function $f(x) = x$; $[-\pi, \pi]$ (03)

Hence show that $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots$

(B) Find the fourier expansion of $f(x) = \begin{cases} 0 & ; 0 \leq x \leq \pi \\ 1 & ; \pi \leq x \leq 2\pi \end{cases}$ (04)

(C) Find a Fourier series for the f^{ns} $f(x) = x^2 - 2$; $-2 < x < 2$. (04)

OR

Que-2 11

(A) Prove that for $-\pi < x < \pi$; (03)

$\sin ax = \frac{2\sin a\pi}{\pi}\left[\frac{\sin x}{1-a^2} - \frac{2\sin 2x}{2^2-a^2} + \frac{3\sin 3x}{3^2-a^2} - \ldots\ldots\ldots\right]$

(B) Obtain a Fourier series for the f^{ns} $f(x)$ defined as :

$$f(x) = \begin{cases} -x-\pi \ ; & -\pi \le x \le 0 \\ x-\pi \ ; & 0 \le x \le \pi \end{cases}$$

(C) Obtain Half range Co – sine series for $f(x) = \begin{cases} 0 \ ; & 0 \le x \le \pi/2 \\ \pi/2 \ ; & \pi/2 \le x \le \pi \end{cases}$

Que-3 **Attempt Any Two**

(A) Find the Fourier transform of $f(x) = \begin{cases} 1 \ ; & |x| < 1 \\ 0 \ ; & |x| > 1 \end{cases}$.

Hence evaluate $\displaystyle\int_0^\infty \frac{\sin x}{x}\, dx$

(B) Find the Fourier Cosine transform of : $e^{-2x} + 4e^{-3x}$.

(C) Find the Fourier Sine transform of : $f(x) = e^{-|x|}$.

Hence show that : $\displaystyle\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi e^{-m}}{2}$; $m > 0$.

Section – II

(12)

Que-4

(A) Solve : $[D^3 + 2D^2 + D]\, y = e^{2x} + x^2 + x$.

(B) Solve : $[D^2 - 4D + 3]\, y = e^x \cos 2x$.

(C) Solve Cauchy's homogeneous equation : $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = \log x$.

OR (12)

Que-4

(A) Solve : $[D^4 - 1]\, y = \cos x \cosh x$.

(B) Solve : $[D^2 - 2D + 3]\, y = x^2 + \cos x$.

(C) Solve Cauchy's homogeneous equation : $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 5y = x^2 \sin(\log x)$.

Que-5 (03

(A) Determine analytic function whose Imaginary part is : $e^x(x\cos y - y \sin y)$ (04

(B) Find the bilinear transformation which maps the points $Z = 1, i, -1$ onto the points $W = i, 0, -i$. (04

(C) Evaluate : $\displaystyle\oint_C \frac{e^{-2z}}{(z+1)^3}\, dz$; where C is the circle : $|z| = 3$.

OR

Que-5

(A) If $f(z) = u + iv$ is an analytic function of z then find $f(z)$ if $u - v = (x-y)(x^2 + 4xy + y^2)$.

(B) Find the bilinear transformation which maps the points $Z = 2, 1, 0$ onto the points $W = 1, 0, i$.

(C) Evaluate : $\displaystyle\int_0^{2+i} (\bar{z})^2\, dz$; Along (1) The line $y = \dfrac{x}{2}$ and (2) The real axis to 2 and then vertically to $(2+i)$.

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6 **Attempt Any Two**

(A) Apply the method of variation of parameters to solve : $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$.

(B) Verify Cayley Hamilton theorem and apply it to find inverse for given matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

(C) Check whether the given matrix is diagonal or not : $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

End of Paper