Seat No:________

GANPAT UNIVERSITY

Bachelor Of Technology (MR) Semester- IV CBCS Regular Examination-May-2014

Sub: 2MR403 - MATHS III - Theory

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Question-1 Attempt the following. (12)

(A) Prove that: (1) $L\{\sin at\} = \dfrac{a}{s^2+a^2}\ ; s>0$ (2) $L\{1\} = \dfrac{1}{s},\ \ s>0$

(B) State convolution theorem and using it evaluate: $L^{-1}\left\{\dfrac{1}{(s+1)(S^2+1)}\right\}$

(C) If $L\{f(t)\} = \overline{f}(s)$, prove that $L\left\{\dfrac{f(t)}{t}\right\} = \int_s^{\infty} L\{f(t)\}ds$, hence find $L\left\{\dfrac{1-e^{at}}{t}\right\}$

Question-1

OR (12)

(A) Define unit step function. Transfer the function $f(t) = \begin{cases} 0 & ; \ 0 \le t \le \pi \\ t & ; \ t \ge \pi \end{cases}$ in to unit step function and find its Laplace transform.

(B) Express the function $f(x) = \begin{cases} -e^{3x} & , x<0 \\ e^{-3x} & , x>0 \end{cases}$ as a Fourier integral and hence show that

$$\int_0^{\infty} \frac{\lambda . \sin \lambda x}{\lambda^2+9} d\lambda = \frac{\pi}{2} e^{-3x} \ \ if \ \ x>0$$

(C) If $L\{f(t)\} = \overline{f}(s)$, prove that $L\{t^n . f(t)\} = (-1)^n \dfrac{d^n}{ds^n} L\{f(t)\}$

Question-2 Attempt the following.

(A) Show that $0 \le x \le \pi$, the sine series for $x(\pi - x)$ is $\dfrac{8}{\pi}\left(\dfrac{sinx}{1^3} + \dfrac{sin3x}{3^3} + \dfrac{sin5x}{5^3} + ...\right)$ (03)

(B) Obtain a Fourier series to represent a function defined by $f(x) = \begin{cases} -\pi & ; \ -\pi \le x \le 0 \\ x & ; \ 0 \le x \le \pi \end{cases}$.

Hence deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + ...$ (04)

(C) Find a Fourier series expansion to represent a function $f(x) = x - x^2 \ ; -\pi \le x \le \pi$ (04)

Question-2

OR

(A) Prove that for $0 \le x \le 2\pi$, (04)

$$e^{ax} = \frac{e^{2\pi a} - 1}{\pi a}\left[\frac{1}{2} + \left\{\frac{a^2}{a^2+1^2}cosx + \frac{a^2}{a^2+2^2}cos2x + ...\right\} - \left\{\frac{a}{a^2+1^2}sinx + \frac{2a}{a^2+2^2}sin2x + ...\right\}\right]$$

(B) Find a Fourier series to represent a function $f(x)$ defined by $f(x) = \begin{cases} x & ; \ 0 \le x \le 1 \\ 2-x & ; 1 \le x \le 2 \end{cases}$ (04)

(C) Find a Fourier cosine series to represent $f(x) = \pi - x \ ; \ 0 \le x \le \pi$ (03)

Question-3 Attempt the any three. (12)

(A) Evaluate: (1) $L\{e^{2t} . t . sint\}$ (2) $L^{-1}\left\{\dfrac{1}{s^2+2s+5}\right\}$

(B) *Use Laplace transform method to solve:* $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=0$ *, where* $y(0)=0, y'(0)=1$

(C) *Find Fourier integral representation of the function* $f(x)=\begin{cases}1, & |x|\prec 1\\ 0, & |x|\succ 1\end{cases}$ *and hence evaluate*

$$(1)\int_0^{\infty}\frac{\sin\lambda.\cos\lambda x}{\lambda}\,d\lambda \quad \text{and} \quad (2)\int_0^{\infty}\frac{\sin\lambda}{\lambda}\,d\lambda$$

(D) *Use partial fraction method to evaluate* $L^{-1}\left\{\frac{1}{(s+1)(s-1)^2}\right\}$

Section-II

Question-4 Attempt the following.

(A) *Units of Population are: 10, 12, 20, 22, 26. How many samples of size 2 without replacement can be drawn from it? Making a list of all samples & verify following results.* (i) $E(\bar{x})=y$ (ii) $E(s^2)=S^2$ **[4]**

(B) (i) *Find* $(250)_{10}=(?)_2$ **[4]**
(ii) *Define: Statistics and Parameter*

(C) *Prove.* (i) $(x+y)'=x'\cdot y'$ (ii) $(x\cdot y)'=x'+y'$ **[4]**

Question-4

OR

(A) *Give the difference between population survey and sample survey.* **[4]**

(B) (i) *Find* $(150)_{10}=(?)_8$ **[4]**
(ii) *Sampling with replacement and without replacement*

(C) *Convert* $f(A,B,C)=A+B'C$ *in to sum of min − terms.* **[4]**

Question-5 Attempt the following.

(A) *Evaluate y when x = 82, using forward interpolation formula for below data.* **[4]**

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

(B) *Evaluate* $\int_0^1 \frac{dx}{1+x}$ *with h = 0.2 using (i) Trapezoidal (ii) Simpsons'* $\frac{1}{3}$ *rule.* **[4]**

(C) *In usual notation prove that;* (i) $\Delta=\nabla E$ (ii) $E=e^{hD}$ **[3]**

Question-5

OR

(A) *Using Newton's formula find Pressure when temperature is 142°C.* **[4]**

T(Tempera	140	150	160	170	180
P(Pressur	3.685	4.854	6.302	8.076	10.225

(B) *Use Lagrange's formula to obtain y when x = 10* **[4]**

x	5	6	9	11
y	12	13	14	16

(C) *Evaluate* $\Delta^2(4\cdot 5^{3x-2})$ **[3]**

Question-6 Attempt the any three.

(A) *Calculate the co − efficient of correlation for the given data.* **[4]**

x	9	8	7	6	5	4	3	2	1
y	15	16	14	13	11	12	10	8	9

(B) *Calculate the Spearman's rank co − efficient for the following data.* **[4]**

x	39	65	62	90	82	75	25	98	36	78
y	47	53	58	86	62	68	60	91	51	84

(C) *Obtain two regression lines for the following data.* **[4]**

x	43	44	46	40	44	42	45	42	38	40	42	57
y	29	31	19	18	19	27	27	29	41	30	26	10

(D) *Explain: (i) Types of Correlation (ii) Scatte Diagram method* **[4]**