

Time: 3 hrs

Total marks: 60

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I**Question-1 Attempt the following.**(A) Find Fourier series expansion of $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$ (04)(B) Find Fourier series of $f(x) = |x|, -2 < x < 2$. (03)(C) Find half range cosine series for $f(x) = (x - 1)^2, 0 < x < 1$ (03)**OR**(A) Expand $e^{ax}, -\pi < x < \pi$ as Fourier series. (04)(B) Find Fourier series for x^2 in the interval $-\pi < x < \pi$. (03)Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$ (C) If $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ show that $f(x) = \frac{4}{\pi} \left(\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \dots \right)$ (03)**Question-2 Attempt the following.**(A) Solve $\frac{d^2y}{dx^2} - 4y = xe^x$ (04)(B) Using variation of parameters method, solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ (03)(C) Solve Cauchy homogeneous equation $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 \sin(\ln x)$ (03)**OR**(A) Solve simultaneous equations $\frac{dx}{dt} = y + 1, \frac{dy}{dt} = x + 1$ (04)(B) Solve $(D^2 + 2D - 3)y = e^x + 4e^{2x}$ (03)(C) Find particular integral (P.I.) of $\frac{d^3y}{dx^3} - \frac{dy}{dx} = x^3$ (03)**Question-3 Attempt the following.**

(A) Give difference between population study and sample study. (10)

(B) For studying a characteristic the observations of a population are 2, 5, 8, and 9. How many samples of size 2 with replacement can be taken from it? Making a list of all the samples and: (I) verify the result $E(\bar{x}) = \bar{y}$ and (II) calculate $Var(\bar{x})$.

(C) What are the characteristic of good sample?

Question-4 Attempt the following.

- (A) Using square formula compute regression line of
- y
- on
- x
- from the following data. (04)

x	23	27	28	29	30	31	33	35	36	39
y	18	22	23	24	25	26	28	29	30	32

- (B) Apply Taylor's series method solve
- y
- at
- $x=1.1$
- and
- $x=1.2$
- if
- $y' = x^2 + y^2$
- ,
- $y(1) = 2.3$
- . (03)

- (C) Use Newton's interpolation formula to find the value of
- y
- for
- $x=1.4$
- and
- $x=1.8$
- from the following data. (03)

x	1.1	1.3	1.5	1.7
y	0.21	0.69	1.25	2.61

OR

- (A) Obtain the equation of regression line of
- x
- on
- y
- of the form
- $x - \bar{x} = b_{xy}(y - \bar{y})$
- (04)

- (B) Using difference formula compute regression line of
- x
- on
- y
- from the following data. (03)

x	91	97	108	121	67	124	51	73	111	57
y	71	75	69	97	70	91	39	61	80	47

- (C) Use Picard's method to solve
- $\frac{dy}{dx} = x + y^2$
- ,
- $y(0) = 1$
- up to second approximation. (03)

Question-5 Attempt the following.

- (A) Derive Karl Pearson's rank correlation coefficient
- $r = \frac{n\sum xy - \sum x \cdot \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$
- (04)

- (B) Calculate the rank correlation coefficient for the following data using Spearman's formula (03)

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

- (C) Define probable error in Bivariate distribution and Calculate it for the following distribution. (03)

x	1	2	3	4	5	6	7
y	8	6	10	12	14	16	18

OR

- (A) Use fourth order R-K method to solve
- $\frac{dy}{dx} = xy + y^2$
- ,
- $y(0) = 1$
- to compute
- $y(0.2)$
- with
- $h=0.2$
- . (04)

- (B) For the two independent variables
- x
- and
- y
- if
- $\sigma_x = 6$
- and
- $\sigma_y = 4$
- , find the correlation coefficient between
- $x + y$
- and
- $x - y$
- . (03)

- (C) Prove that correlation coefficient is independent of shift origin and change of scale. (03)

Question-6 Attempt the following.

- (A) Using Lagrange's interpolation formula find second degree polynomial which passes through the following points. (04)

x	1	3	7
y	12	16	48

- (B) Attempt any Two. (06)

- (a) Derive the equation of regression line of
- y
- on
- x
- of the form
- $y - \bar{y} = b_{yx}(x - \bar{x})$

- (b) Use Euler's method to solve differential equation
- $\frac{dy}{dx} = x + y + xy$
- ,
- $y(0) = 1$
- to calculate
- $y(1)$
- with
- $h=0.2$

- (c) Evaluate:
- $\int_0^1 \frac{1}{1+x^2} dx$
- using (1) Simpson's 1/3 rule and (2) Trapezoidal rule.

END OF PAPER