Seat No:__________

GANPAT UNIVERSITY

Bachelor of Technology (M.E) Semester- IV CBCS Regular Examination-May-2014

Sub: 2HS401 - Engineering Mathematics - III - Theory

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Question-1 Attempt the following. (12)

(A) Prove that: (1) $L\{\sin at\} = \dfrac{a}{s^2+a^2}\ ; s>0$ (2) $L\{e^{at}\} = \dfrac{1}{s-a}\ ,\ s>a$

(B) State convolution theorem and using it evaluate: $L^{-1}\left\{\dfrac{1}{(S^2+a^2)^2}\right\}$

(C) If $L\{f(t)\} = \overline{f}(s)$, prove that $L\{t^n.f(t)\} = (-1)^n \dfrac{d^n}{ds^n} L\{f(t)\}$

OR (12)

(A) Define unit step function. Transfer the function $f(t) = \begin{cases} 0 & ; 0\le t\le 1 \\ 2 & ; t\ge 1 \end{cases}$ in to unit step function and find its Laplace transform.

(B) Express the function $f(x) = \begin{cases} -e^{kx}, & x<0 \\ e^{-kx}, & x>0 \end{cases}$ as a Fourier integral and hence show that

$$\int_0^\infty \frac{\lambda.\sin\lambda x}{\lambda^2+k^2}\, d\lambda = \frac{\pi}{2} e^{-kx} \ \text{if}\ x>0, k>0$$

(C) If $L\{f(t)\} = \overline{f}(s)$, prove that $L\left\{\dfrac{f(t)}{t}\right\} = \displaystyle\int_s^\infty L\{f(t)\}\, ds$, hence find $L\left\{\dfrac{1-e^{at}}{t}\right\}$

Question-2 Attempt the following.

(A) Find a Fourier series expansion to represent a function $f(x) = x - x^2\ ; -\pi \le x \le \pi$ (04)

(B) Obtain a Fourier series to represent a function defined by $f(x) = \begin{cases} -\pi & ; -\pi\le x\le 0 \\ x & ; 0\le x\le \pi \end{cases}$. (04)

Hence deduce that $\dfrac{\pi^2}{8} = \dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dots\dots\dots$

(C) Show that $0 \le x \le \pi$, the sine series for $x(\pi - x)$ is $\dfrac{8}{\pi}\left(\dfrac{\sin x}{1^3} + \dfrac{\sin 3x}{3^3} + \dfrac{\sin 5x}{5^3} + \dots\dots\dots\right)$ (03)

Question-2 OR

(A) Find a Fourier series to represent a function $f(x)$ defined by $f(x) = \begin{cases} \dfrac{x}{l} & ; 0\le x\le l \\ 2-\dfrac{x}{l} & ; l\le x\le 2l \end{cases}$ (04)

(B) Prove that for $0 \le x \le 2\pi$, (04)

$$e^{ax} = \frac{e^{2\pi a}-1}{\pi a}\left[\frac{1}{2} + \left\{\frac{a^2}{a^2+1^2}\cos x + \frac{a^2}{a^2+2^2}\cos 2x + \dots\right\} - \left\{\frac{a}{a^2+1^2}\sin x + \frac{2a}{a^2+2^2}\sin 2x + \dots\right\}\right]$$

(C) Find a Fourier cosine series to represent $f(x) = \pi - x\ ; 0 \le x \le \pi$ (03)

Question-3 Attempt the any three. (12)

(A) Find Fourier integral representation of the function $f(x)=\begin{cases}1, & |x|<1\\ 0, & |x|>1\end{cases}$ and hence evaluate

(1) $\int_0^\infty \frac{\sin\lambda.\cos\lambda x}{\lambda}d\lambda$ and (2) $\int_0^\infty \frac{\sin\lambda}{\lambda}d\lambda$

(B) Evaluate: (1) $L\{e^t.t.\cos t\}$ (2) $L^{-1}\left\{\frac{1}{s^2+2s+10}\right\}$

(C) Use Laplace transform method to solve: $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=0$, where $y(0)=1, y'(0)=0$

(D) Use partial fraction method to evaluate $L^{-1}\left\{\frac{1}{(s+1)(s-1)^2}\right\}$

SECTION-II

Question-4 (12)

(A) Find the analytic function $f(z)=u+iv$ for which $u+v=e^x(\cos y+\sin y)$.

(B) State Cauchy's Theorem and evaluate $\oint \frac{e^{3z}}{(z-1)(z-2)}dz$, *where* $C: |z|=3$

(C) If $w=T_1(z)=\frac{z-2}{z+3}$ and $w=T_2(z)=\frac{z}{z+2}$ then find $T_1^{-1}(w)$, $T_2^{-1}(w)$ and $T_1\cdot T_2$

OR

Question-4 (12)

(A) Find fixed points, normal form and decide the type of transformation for $w=\frac{5z+4}{z+5}$

(B) Discuss the analyticity of the function $f(z)=\frac{1}{z}$ and find its derivative.

(C) Evaluate $\oint |z|^2\,dz$, around the square with vertices (0 ,0) , (1 ,0) , (1 ,1) and (0 ,1).

Question-5 (11)

(A) Form PDE by eliminating arbitrary function and constant from the equations,
(1) $z=f(x^2+y^2)$ (2) $z=A\cdot e^{pt}\sin px$

(B) Solve: $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$ by method of multipliers.

(C) Form Partial Differential Equation from $\emptyset\,(x^2+y^2+z^2, lx+my+nz)=0$

OR

Question-5 (11)

(A) Solve Partial Differential Equation by method of separable variables $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$, with $u(x,0)=3e^{-y}-e^{-5y}$

(B) Solve $\frac{\partial^2 z}{\partial x^2}=z$, given that $x=0, z=e^y$ and $\frac{\partial z}{\partial x}=e^{-y}$.

Question-6 Attempt the any three (12)

(A) Apply Variation of Parameter to solve $(D^2+1)y=\operatorname{cosec} x$

(B) Solve this cauchy'sHomogeneous linear equation: $u=r\frac{d}{dr}\left[r\frac{du}{dr}\right]+ar^3$

(C) Solve: (1) $(D^2+4)y=e^x\sin x$ (2) $(D^2+5D+4)y=e^{2x}$

(D) Solve: $(x+1)^2\frac{d^2y}{dx^2}+(x+1)\frac{dy}{dx}+y=2\cos[\log(1+x)]$

End of Paper