

02/11/16
Date: 03/05/2016

Exam No: _____

GANPAT UNIVERSITY

B.TECH. SEM- IV (ME-MC) REGULAR EXAMINATION APRIL-JUNE 2016

2HS402: Mathematics for Mechanical & Mechatronics Engineering

Time: 3 HRS.

Total Marks: 60

Instructions:

- (1) This Question paper has two sections. Attempt each section in separate answer book.
- (2) Figures on right indicate marks.
- (3) Be precise and to the point in answering the descriptive questions.

SECTION: I

Question: 1

(A) Evaluate: (i) $L \{2t^3 + e^{-2t} + t^{4/3}\}$ (ii) $L \{e^{-t} \sin^2 2t\}$ [4]

(B) Find $L^{-1} \left\{ \frac{s}{s^4 + 4a^4} \right\}$ [3]

(C) Evaluate $\int_0^\infty \frac{e^{-t} \sin^2 t}{t} dt$ using laplace transform method. [3]

OR

Question: 1

(A) Find laplace transform of $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$ [4]

(B) Apply convolution theorem to find $L^{-1} \left\{ \frac{1}{(s+1)(s+3)} \right\}$ [3]

(C) Solve the differential equation $\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} = -8t$; $y(0) = 0$, $y'(0) = 0$ [3]

Question: 2

(A) Obtain fourier series for $f(x) = x$, $0 < x < 2\pi$ hence deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \dots$ [4]

(B) Develop fourier series for $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ [3]

(C) If $f(x) = x^2$, $0 < x < 2$ then find half range sine series. [3]

OR

Question: 2

(A) Expand $f(x) = e^x$ as a fourier series in the interval $(0, 1)$ [5]

(B) Find fourier series for $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$ and $f(x + \pi) = f(x)$ [5]

Question: 3 Attempt any Two

- (A) Express the function $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ as a fourier integral and hence evaluate [5]

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda \text{ and } \int_0^{\infty} \frac{\sin x}{x} dx$$

- (B) Evaluate: (i) $L^{-1} \left\{ \log \left(\frac{s+1}{s-1} \right) \right\}$ (ii) $L \{ t^2 \cdot \sin^2 t \}$ [5]

- (C) Find fourier series to represent $\cosh ax$ between $-\pi$ to π . [5]

SECTION: II

Question: 4

- (A) Solve: (i) $(D^2 - 5D + 6)y = \sin 3x$ (ii) $(D^3 - D^2 - 6D)y = x^2 + 1$ [4]

- (B) Apply method of variation of parameters to solve $y'' + a^2y = \sec ax$ [3]

- (C) Solve the initial value problem $y'' + y' - 2y = 0$, $y(0) = 4$ and $y'(0) = -5$ [3]

OR

Question: 4

- (A) Solve: $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ [5]

- (B) Solve the following simultaneous differential equations.

$$\frac{dx}{dt} - \frac{dy}{dt} - y = -e^t, \quad x - y + \frac{dy}{dt} = e^{2t} \quad [5]$$

Question: 5

- (A) Check the analiticity of functions (i) $f(z) = \sin z$ (ii) $f(z) = z^{3/2}$ [4]

- (B) Find fixed points, normal forms & decide the type of transformation $w = \frac{2z-5}{z-4}$ [3]

- (C) Find the analytic function whose real part is $e^{2x} (x \cos 2y - y \sin 2y)$ [3]

OR

Question: 5

- (A) Evaluate $\int_C |z| dz$ along the sides of square with vertices $(0,0), (1,0), (1,1)$ & $(0,1)$ [4]

- (B) Evaluate $\int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z-2)} dz$ where $C: |z|=2$ [3]

- (C) Check whether the given function is harmonic or not. $u(x,y) = y + e^x \cos y$ [3]

Question: 6 Attempt any Two

- (A) Solve: $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$ [5]

- (B) Form PDE by eliminating arbitrary function and arbitrary constant. [5]

$$(i) f(x^2 + y^2 + z^2, lx + my + nz) = 0 \quad (ii) 2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- (C) (I) Solve: $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x$ [3]

- (II) If $P(A) = 0.3, P(B) = 0.78$ & $P(A \cap B) = 0.16$ then find $P(A \cup B)$ & $P(A' \cap B')$ [2]