Seat No.______

GANPAT UNIVERSITY

B.TECH. (MC) SEM – IV CBCS REGULAR THEORY EXAMINATION

SUBJECT: 2HS401 MATHEMATICS – III (Common with ME)

MAY – JUNE 2012

TIME: - 3 HOURS **TOTAL MARKS: 70**

INSTRUCTIONS:

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

SECTION – I

Question-1 Attempt the following: **(12)**

(A) Derive $L\{\cos at\}$ and Evaluate: $L\left\{\dfrac{Sint}{t}\right\}$

(B) Evaluate: (1) $L\left\{(t+2)^2 e^t\right\}$ (2) $L^{-1}\left\{\dfrac{1}{S^4 - 2S^3}\right\}$

(C) Solve differential equation $y''' + 2y'' - y' - 2y = 0$, $y(0) = y'(0) = 0$, $y''(0) = 6$

OR

Question-1 **(12)**

(A) Derive $L\{e^{at}\}$ and Evaluate: $L^{-1}\left\{\log\left(\dfrac{S^2+1}{S^2}\right)\right\}$

(B) If $L\{f(t)\} = \bar{f}(s)$ then $L\left\{\int_0^t f(u)\,du\right\} = \dfrac{\bar{f}(s)}{s}$

(C) Evaluate : $L\{sint.u(t-\pi)\}$

Question-2 Attempt the following:

(A) Find a Fourier series to represent : $f(x) = x^2$, $-\pi < x < \pi$ **(03)**

Hence deduce : $\dfrac{\pi^2}{12} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + - - -$

(B) Find a Fourier series for the function : $f(x) = \begin{cases} 1 + \dfrac{2x}{\pi} & ; \quad -\pi < x < 0 \\ 1 - \dfrac{2x}{\pi} & ; \quad 0 < x < \pi \end{cases}$ **(04)**

(C) Expand : $f(x) = \pi x - x^2$ in a half range sine series in the interval $(0, \pi)$ **(04)**

OR

Question-2

(A) Find a Fourier series for the function: $f(x) = e^{-x}$, $[-2, 2]$ **(03)**

(B) Find a Fourier series to represent the function $f(x) = \begin{cases} -1 & ; \quad -\pi \le x \le 0 \\ 1 & ; \quad 0 \le x \le \pi \end{cases}$ **(04)**

Hence deduce : $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + - - -$

(C) Find the half range cosine series to represent $f(x) = x^2$, $0 \le x \le \pi$ **(04)**

Question-3 Attempt any three: **(12)**

(A) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & ; \quad |x| \le 1 \\ 0 & ; \quad |x| > 1 \end{cases}$

(B) Find Fourier sine transform of $e^{-|x|}$. Hence show that: $\int_0^{\infty} \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}$; $m > 0$

(C) State & Prove convolution theorem.

(D) A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively, what is the probability that the problem will be solved.

Section – II

Question-4 Attempt the following: **(12)**

(A) Solve : $(D^2 + 3D + 2)y = e^{-x}$

(B) Solve : $(D^3 - 3D^2 + 9D - 27)y = e^x + \cos x$

(C) Solve by the method of variation parameters: $y'' - 6y' + 9y = e^{3x} / x^2$

Question-4 **OR** **(12)**

(A) Solve : $(D^2 + 3)y = x^2 \cos x$

(B) Solve : $\frac{d^3 y}{dx^3} - \frac{dy}{dx} = x^3$

(C) Solve the Cauchy-Euler differential equation : $x^2 \frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

Question-5 Attempt the following: **(12)**

(A) Solve : $x^2 p + y^2 q = (x + y)z$

(B) Solve : $(y + z)p + (z + x)q = (x + y)$

(C) Using method of separation of variables, solve the differential equation $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0, y) = e^{-5y}$

Question-5 **OR** **(12)**

(A) Solve : $y^2 z \frac{\partial z}{\partial x} - x^2 z \frac{\partial z}{\partial y} = x^2 y$

(B) Solve : $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$

(C) Solve the equation, by method of separation of variables $u_{xy} = -u_x$.

Question-6 Attempt the following:

(A) Determine whether the function $f(z) = \begin{cases} \frac{z^2 + 3iz - 2}{z + i} & z \neq -i \\ 5 , & z = -i \end{cases}$ is continuous? **(03)**

(B) Find analytic function whose real part is $u = e^{-x}(x\cos y + y\sin y)$ **(04)**

(C) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y), but are not harmonic conjugates **(04)**

END OF PAPER