Seat No.________

GANPAT UNIVERSITY

B. TECH. SEM. IV CBCS (ME/MC) EXAMINATION. MAY – 2013

Sub : (2HS 401) Mathematics – III

Time: 3 hrs Total marks: 70

Instruction : (1) All questions are compulsory
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Que 1 Answer the following. (12)

(a) Determine the f^{ns} : $\frac{x}{x^2+y^2} + i\frac{y}{x^2+y^2}$ is Analytic or not .

(b) Find the Bilinear transformation which maps the points $Z = 2, i, -2$ onto $W = 1, I, -1$

(c) State Cauchy's integral formula and Evaluate

$$\int_C \frac{e^{2z}}{(z-1)(z-2)}\,dz \quad ; \text{ where C is the circle } |z| = 3$$

OR

Que 1 Answer the following. (12)

(a) If f(z) is Analytic f^{ns} of z then P.T. $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$.

(b) Evaluate : $\int_0^{1+i} (x - y + ix^2)\,dz$; along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$

(c) If $f(z) = u + iv$ is an Analytic f^{ns} of z then find f(z) if

$$u - v = e^x\,(\cos y - \sin y)$$

Que 2 Answer the following.

(a) Form a partial differential equation by eliminating arbitrary constant or Function from (i) $z = (x+a)(y+b)$ (ii) $z = f(x^2 + y^2)$ (03)

(b) Solve : $(y^2 z)p + (x^2 z)q = xy^2$ (04)

(c) Solve by the method of separation of variables : $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ (04)

OR

Que 2 Answer the following.

(a) Form a partial differential equation by eliminating arbitrary Function from

$f(xy + z^2, x + y + z) = 0$

(b) Solve : Solve : $x^2 p + y^2 q = (x + y) z$ (04

(c) Solve : $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$; for which $\dfrac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ AND (04

$z = 0$ when y is an odd multiple of $\dfrac{\pi}{2}$

Que 3 Attempt any three. (12

(a) Solve : $[D - 2]^2 y = e^{2x} + \sin 2x + x^2$

(b) Apply the method of variation of parameters to solve : $\dfrac{d^2y}{dx^2} + y = \sec x$

(c) Solve Cauchy's homogeneous equation : $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} = 12 \log x$

(d) Solve : $(D^2 - 2D + 1) y = x \sin 2x$

Section – II

Que 4 Answer the following. (12

(a) Find (1) $L\{e^{5t} + \cos 3t - e^{5t} \cos 3t\}$ (2) $L^{-1}\left\{\log\left(\dfrac{s+1}{s-1}\right)\right\}$

(b) Evaluate (1) $L\left\{\dfrac{e^{at} - \cos at}{t}\right\}$ (2) $L^{-1}\left\{\dfrac{6s-7}{s^2+5}\right\}$

(c) Solve using the Laplace transforms $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 6$

OR

Que 4 Answer the following. (12)

(a) Find the Laplace transform of $f(t) = \begin{cases} 0 & 0 \le t \le 2 \\ 3 & t \ge 2 \end{cases}$

(b) Evaluate (1) $L\{t^2 \sin 4t\}$ (2) $L^{-1}\left\{\dfrac{1-3s}{s^2+8s+21}\right\}$

(c) Evaluate $\displaystyle\int_0^{\infty} \frac{e^{-t} \sin^2 t}{t} dt$ using Laplace transforms

Que 5 Answer the following.

(a) Express $f(x) = |x|$, $-\pi \le x \le \pi$ as Fourier series. (04)

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(b) Find a Fourier series to represent: $f(x)=\begin{cases}-\pi & -\pi<x<0\\ x & 0<x<\pi\end{cases}$ (04)

(c) Find the half range Cosine series for $f(x)=(x-1)^2;\ 0<x<1$ (03)

OR

Que 5 Answer the following.

(a) Find a Fourier series to represent for $f(x)=e^{-x}\ ,0<x<2\pi$, (04)

(b) Find a Fourier series to represent: $f(x)=\begin{cases}x & 0<x<1\\ 1-x & 1<x<2\end{cases}$ (04)

(c) Express $\sin x$ as a cosine series in $0<x<\pi$ (03)

Que 6 Attempt any three. (12)

(a) State and prove convolution theorem.

(b) Find the Laplace transform of the periodic function

$$f(t)=\frac{t}{2},\ 0<t<3\ with\ f(t+3)=f(t)$$

(c) Express the following function in terms of unit step function and find its Laplace transforms $f(t)=\begin{cases}t-1 & 1<t<2\\ 3-t & 2<t<3\end{cases}$

(d) Using Fourier sine integral show that $\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda\, d\lambda=\begin{cases}\frac{1}{2}\pi & when\ 0<x<\pi\\ 0 & when\ x>\pi\end{cases}$

END OF PAPER

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