Seat NO.______________

GANPAT UNIVERSITY

Bachelor of Technology (M.C) Semester- IV CBCS Regular Examination-May-2014

Sub: 2HS401 - Engineering Mathematics - III - Theory

Time: 3 hrs **Total marks: 70**

Instruction: (1) All questions are compulsory.

(2) Write answer of each section in separate answer books.

(3) Figures to the right indicate marks of questions.

Section – I

Question-1 Attempt the following. (12)

(A) Prove that: (1) $L\{1\} = \frac{1}{s}$, $s > 0$ (2) $L\{cosat\} = \frac{s}{s^2+a^2}$; $|s|>|a|$

(B) State convolution theorem and using it evaluate: $L^{-1}\left\{\frac{1}{(S-1)(s^2+1)}\right\}$

(C) Use Laplace transform method to solve: $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$, where $y(0) = 0, y'(0) = 1$

Question-1 OR (12)

(A) Define unit step function. Transfer the function $f(t) = \begin{cases} 0 & , 0 \le t \le \pi \\ sint & , t \ge \pi \end{cases}$ in to unit step function and find its Laplace transform.

(B) Express the function $f(x) = \begin{cases} -e^{x} & , x < 0 \\ e^{-x} & , x > 0 \end{cases}$ as a Fourier integral and hence show that $\int_0^\infty \frac{\lambda . \sin \lambda x}{\lambda^2+1} d\lambda = \frac{\pi}{2} e^{-x}$ if $x > 0$

(C) If $L\{f(t)\} = \bar{f}(s)$, prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty L\{f(t)\} ds$, hence find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$

Question-2 Attempt the following.

(A) Using sine series, Show that $x(\pi - x) = \frac{8}{\pi}\left(\frac{sinx}{1^3} + \frac{sin3x}{3^3} + \frac{sin5x}{5^3} + \dots\dots\dots\right)$, where $0 \le x \le \pi$ (03)

(B) Obtain a Fourier series to represent a function defined by $f(x) = \begin{cases} -\pi & ; -\pi \le x \le 0 \\ x & ; 0 \le x \le \pi \end{cases}$. (04)

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots$

(C) For $-\pi \le x \le \pi$, prove that (04)

$$x + x^2 = \frac{\pi^2}{3} - 4\left\{\frac{cosx}{1^2} - \frac{cos2x}{2^2} + \frac{cos3x}{3^2} \dots\dots\right\} + 2\left\{\frac{sinx}{1} - \frac{sin2x}{2} + \frac{sin3x}{3} \dots\dots\right\}.$$

Question-2 OR

(A) Find a Fourier sine series to represent $f(x) = \pi x - x^2$; $0 \le x \le \pi$ (03)

(B) Find a Fourier series to represent a function $f(x)$ defined by $f(x) = \begin{cases} 0 & ; -\pi \le x \le 0 \\ 1+x & ; 0 \le x \le \pi \end{cases}$ (04)

(C) Prove that for $-\pi \le x \le \pi$, $coshax = \frac{2a}{\pi} sinha\pi \left[\frac{1}{2a^2} + \sum_{n=1}^{\infty} \frac{(-1)^n cosnx}{n^2+a^2}\right]$ (04)

Question-3 Attempt the any three. **(12)**

(A) Find Fourier integral representation of the function $f(x)=\begin{cases}1\ , & |x|\prec 1\\ 0\ , & |x|\succ 1\end{cases}$ and hence evaluate

(1) $\int_0^{\infty}\frac{\sin\lambda.\cos\lambda x}{\lambda}\,d\lambda$ and (2) $\int_0^{\infty}\frac{\sin\lambda}{\lambda}\,d\lambda$

(B) Evaluate: (1) $L\{\,t.\sin 2t\}$ (2) $L\{\,e^{t}(t+2)^{2}\}$

(C) If $L\{f(t)\}=\overline{f}(s)$, prove that $L\{t^{n}.f(t)\}=(-1)^{n}.\frac{d^{n}}{ds^{n}}L\{f(t)\}$

(D) Use partial fraction method to evaluate $L^{-1}\left\{\frac{s}{(s+2)(s-3)^{2}}\right\}$

SECTION-II

Question-4 Attempt the following. **(12)**

(A) Discuss the analyticity of the function (i) $f(z)=|z|^{2}$ (ii) $f(z)=z^{2}$

(B) Find fixed points, normal form & decide the type of transformation for $w=\frac{z-1}{z+1}$.

(C) State and prove Cauchy's Theorem for Contour Integration.

Question-4 OR **(12)**

(A) Find the analytic function whose real part is $u=\cos x\cosh y$.

(B) Evaluate $\oint_{C}(z-z^{2})\,dz$, where C is the upper half of the circle $|z|=1$.

(C) Define Harmonic function. If $f(z)=u+iv$ is analytic then show that u and v are Harmonic functions.

Question-5 Attempt the following. **(11)**

(A) Form PDE by eliminating arbitrary function and constant from the equations,

(1) $z=ax^{3}+by^{3}$ (2) $z=(x+y)\cdot f(x^{2}-y^{2})$

(B) Solve: $zp+x=0$ by method of grouping.

(C) Form Partial Differential Equation from $\emptyset\,(x+y+z\,,x^{2}+y^{2}+z^{2})=0$

Question-5 OR **(11)**

(A) Solve: $\frac{\partial^{2}z}{\partial x^{2}}+z=0$, given that when $x=0$, $z=e^{y}$ and $\frac{\partial z}{\partial x}=1$

(B) Solve by method of separation of variables $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$, given when $x=0$ $u(x,y)=3e^{-y}-e^{-5y}$.

Question-6 Attempt the any three. **(12)**

(A) Apply Variation of Parameter to solve $(D^{2}+9)y=\sec 3x$

(B) Solve: $x^{3}\cdot\frac{d^{2}y}{dx^{2}}+3x^{2}\cdot\frac{dy}{dx}+xy=\sin(\log x)$

(C) Solve I.V.P $\frac{d^{2}y}{dx^{2}}+4\frac{dy}{dx}-12y=0$, given that when $x=0, y=0$ and $\frac{dy}{dx}=1$.

(D) Solve: (1) $(D^{4}+4)y=0$ (2) $(D^{2}-1)y=5x^{2}$

End of Paper