Seat No.______________

Ganpat University

B.Tech Semester – IV (ME/MC) Regular Examination April - June 2015

2HS401 ENGINEERING MATHEMATICS - III

Time: 3 hrs. **Marks: 70**

1. *All questions are compulsory.*
2. *Write answer of each section in separate answer books.*
3. *Figures to the right indicate marks of questions.*

Section – I

Que: 1 **[12]**

(A) Obtain Fourier series for $f(x) = x - x^2$ in $-\pi \le x \le \pi$.

Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots$

(B) Obtain Fourier series for $f(x) = \begin{cases} \frac{\pi}{2} + x \ ; -\pi < x < 0 \\ \frac{\pi}{2} - x \ ; \ 0 < x < \pi \end{cases}$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

(C) Obtain Fourier series for $f(x) = e^{-x}$; $0 < x < 2\pi$

OR

Que: 1 **[12]**

(A) Obtain Fourier series for $f(x) = x$ in $[-\pi, \pi]$.

Hence deduce that $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\ldots$

(B) Obtain Fourier series for $f(x) = \begin{cases} l - x \ ; 0 < x < l \\ 0 \ ; \ l \le x \le 2l \end{cases}$.

(C) An alternating current after passing through a rectifier has the form :

$$i(x) = \begin{cases} I_0 \sin x \ ; \ 0 < x < \pi \\ 0 \ ; \ \pi < x < 2\pi \end{cases}$$

I_0 is the maximum current & period is 2π Express $i(x)$ as a Fourier series

Que: 2

(A) *Find :* (i) $L\{e^{-2t}\cos 5t \sin 3t\}$ (ii) $L^{-1}\left\{\dfrac{3s+7}{s^2-2s-3}\right\}$. [04]

(B) *If* $L\{f(t)\} = \overline{f(s)}$ *then Prove that* : $L\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n}\left[\overline{f(s)}\right]$ [03]

(C) *State Convolution theorem and apply it to evaluate* $L^{-1}\left\{\dfrac{1}{s^2(s-1)}\right\}$ [04]

OR

Que: 2

(A) *Find :* (i) $L\{t\,e^{2t}\cos 3t\}$ (ii) $L\left\{\dfrac{\cos at-\cos bt}{t}\right\}$ [04]

(B) *Solve* : $\dfrac{dy}{dt} - 4y = 2e^{2t} + e^{4t}$ *by Laplace transform method where* $y(0) = 0$ [03]

(C) *Define Unit step Function. Express the following function in terms of unit step function and find its Laplace transforms :* $f(t) = \begin{cases} 0 \ ; & t < 5 \\ t \ ; & t \geq 5 \end{cases}$ [04]

Que: 3 ***Attempt any Three*** [12]

(A) *Find the Fourier transform of* $f(x) = \begin{cases} 1 \ ; & |x| < 1 \\ 0 \ ; & |x| > 1 \end{cases}$.

Hence evaluate : $\displaystyle\int_0^{\infty} \frac{\sin x}{x}\,dx$.

(B) *Find the Fourier Co – sine transform of* $f(x) = e^{-2x} + 4e^{-3x}$.

(C) *Obtain Fourier series for* $f(x) = x^2$ *in* $[0, \pi]$.

Hence deduce that $\dfrac{\pi^2}{12} = \dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \ldots\ldots$

(D) *Find :* $L^{-1}\left\{\dfrac{3s+1}{(s-1)(s^2+1)}\right\}$

Section – II

Que: 4 [12]

(A) *Solve:* $p(y^2z) + q(zx^2) = (xy^2)$

(B) *Obtain PDE from* $f(x^2+y^2+z^2, z^2-2xy) = 0$

(C) *Find PDE from following equations by eliminating arbitrary constants.*

(i) $z = ax + by + ab$ (ii) $z = (x^2+a)(y^2+b)$

2/3

OR

Que: 4 [12]

(A) Show that general solution of Lagrange's linear equation is $\emptyset(u, v) = 0$ where $\emptyset$ is arbitrary function and u, v are solution of auxiliary equations.

(B) Solve $z_{xy} = \sin x \sin y$, subject to conditions $z_y = -\sin y$ & $z = 0$ when $x = 0$.

Que: 5

(A) Is the function $w = z^{-1}$ analytic? Justify. [03]

(B) State and prove Cauchy's theorem for contour integration. [04]

(C) Prove $T_2 \cdot T_1$ is bilinear transformation if they are itself bilinear. [04]

OR

Que: 5

(A) Derive $C - R$ equations in polar form. [05]

(B) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$ find $f(z)$ in terms of z. [06]

Que: 6 Attempt any Three [12]

(A) Using Variation of parameter method solve $\frac{d^2y}{dx^2} + a^2y = cosec\ ax$

(B) Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$

(C) Solve simultaneous equations. $\frac{dx}{dt} = 5x + y$, $\frac{dy}{dt} = y - 4x$.

(D) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \cdot \log x$

End of Paper