Seat No._______

Ganpat University

B.Tech. (ME/MC) Sem. IV CBCS Regular Examination APRIL-JUNE 2017

Sub :(2HS402) Mathematics for Mechanical and Mechatronics Engineering

Time: 3 hrs — Total marks: 60

Instruction: (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section – I

Question 1 Answer the following.

(A) Evaluate (I) $L(e^{2t}\sin^2 t)$ (II) $L\left(\frac{\cos at-\cos bt}{t}\right)$. (4)

(B) Using convolution theorem, evaluate $L^{-1}\left(\frac{1}{(s+1)(s^2+1)}\right)$. (3)

(C) Find Laplace transform of $f(t)$ where $f(t)=\begin{cases} t+1, & 0<t<3 \\ 1, & t\geq 3\end{cases}$. (3)

OR

(A) Evaluate $L^{-1}\left(\frac{s}{(s-2)(s-1)^2}\right)$ using partial fraction method. (4)

(B) If $L\{f(t)\}=\bar{f}(s)$ and $\frac{f(t)}{t}$ has Laplace transform, prove that $L\left(\frac{f(t)}{t}\right)=\int_s^\infty \bar{f}(s)\,ds$. (3)

(C) Evaluate $L^{-1}\left(\frac{s-3}{s^2-6s+13}\right)$ (3)

Question 2 Answer the following.

(A) Expand $e^{ax}, -\pi<x<\pi$ as Fourier series. (4)

(B) Find half range cosine series for $f(x)=(x-1)^2, 0<x<1$ (3)

(C) Find Fourier series of $f(x)=|x|, -2<x<2$. (3)

OR

(A) Find Fourier series expansion of $f(x)=\begin{cases} -\pi, 0<x<\pi \\ x-\pi, \pi<x<2\pi\end{cases}$ (4)

(B) Find half range sine series of $\frac{\pi}{2}-x, 0<x<\pi$. (3)

(C) Find Fourier series expansion of $f(x)=\begin{cases} x+\frac{\pi}{2}, -\pi<x<0 \\ \frac{\pi}{2}-x, \quad 0<x<\pi\end{cases}$ (3)

Question 3 Answer the following (attempt any two). (10)

(A) Solve $\frac{d^2y}{dt^2}-6\frac{dy}{dt}+8y=e^{-3t}, y(0)=0, y'(0)=1$ by Laplace transform method.

(B) Find Fourier integral representation of $f(x)=\begin{cases} 1, & -1<x<1 \\ 0, & x\in(-\infty,-1)\cup(1,\infty)\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$.

(C) Find Fourier cosine integral of $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2\end{cases}$.

Section –II

Question 4 Answer the following.

(A) Evaluate $\int_{0}^{2+i} (\bar{z})^2 dz$ along the path $y = \frac{x}{2}$. (4)

(B) Find conformal transformation which maps points $z = i, 1, -i$ onto $w = -i, 1, i$. (3)

(C) Find an analytic function whose real part is $u = x^3 - 3x^2y$. (3)

OR

(A) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$ where C is (I) $|z| = 2$, (II) $|z| = \frac{1}{2}$. (4)

(B) Discuss analyticity of sin z. If analytic, find its derivative. (3)

(C) Evaluate $\int_C (z - z^2)dz$ where C is upper half of the circle $|z| = 1$. (3)

Question 5 Answer the following.

(A) Solve $(D^2 + 2D - 3)y = e^x + 2e^{4x} + \cos 2x$ (4)

(B) Apply the method of variation of parameters to solve: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ (3)

(C) Solve $y^2p - xyq = x(z - 2y)$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ (3)

OR

(A) Using separable of variable method, solve $\frac{\partial^2 u}{\partial t \partial x} = e^{-t} \cos x$ (4)

(B) Solve the simultaneous differential equation: $\frac{dx}{dt} = y + 1, \frac{dy}{dt} = x + 1$ (3)

(C) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \log x$ (3)

Question 6 Answer the following.

(A) Solve Cauchy homogeneous equation $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = x^2 \sin(\ln x)$ (4)

(B) **Attempt any two** (6)

(I) Form a partial differential equation by eliminating arbitrary constant from: $x^2 + y^2 + (z - c)^2 = a^2$

(II) Solve : $\frac{\partial^3 z}{\partial x^2 \partial y} = cos(2x + 3y)$ by direct integration.

(III) In a bolt factory machines A, B and C manufacture 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine A?

END OF PAPER