Student Exam No._____

# GANPAT UNIVERSITY

## B. Tech. Semester: III Open Elective(BMI,CE,CL,EC,EE,IT,MC,ME)

## Regular Examination November – December 2013

## 2OS301 - VECTOR CALCULUS & Z TRANSFORM - Theory

**Time:** 3 Hours **Total Marks: 70**

**Instruction:** 1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

### Section-I

**Que-1** **Answer the following.** 12

(a) Verify the Cayley-Hemilton theorem for matrix $A=\begin{bmatrix}1&2&1\\2&3&1\\5&8&4\end{bmatrix}$

(b) Diagonalise the matrix $\begin{bmatrix}0&1\\3&2\end{bmatrix}$

(c) Define : (1) Hermitian matrix (2) Skew- Hermitian matrix (3) Unitary Matrix

**OR**

**Que-1** **Answer the following.** 12

(a) Using Caley-Hemilton theorem find $A^{-1}$ for matrix $A=\begin{bmatrix}1&1&3\\1&3&-3\\-2&-4&-4\end{bmatrix}$

(b) If $A=\begin{bmatrix}2+i&3&-1+3i\\-5&i&4-2i\end{bmatrix}$, Show that $A^{*}A$ is a Hermitian matrix.

(c) Using Caley-Hemilton theorem find $A^3$ for matrix $A=\begin{bmatrix}1&2&1\\2&3&1\\5&8&4\end{bmatrix}$

**Que-2** **Answer the following.** 11

(a) Using Z-transform Using Z-transform prove that $Z\left(n^p\right)=-z\frac{d}{dz}Z\left(n^{p-1}\right)$, p being a positive integer. 3

(b) Using damping rule find (1) $Z\left(na^n\right)$ (2) $Z\left(n^2a^n\right)$ 4

(c) Find inverse Z-transform of $\frac{2z+1}{(z+2)(z-4)}$ using partial fraction method. 4

**OR**

**Que-2** **Answer the following.** 11

(a) Define Z-transform and derive formula for $Z\left(1\right)$ 3

(b) Find (1) $Z\left(\cos n\theta\right)$ (2) $Z\left(\sin n\theta\right)$ 4

(c) Find inverse Z-transform of $\frac{10z}{(z-1)(z-2)}$ using Convolution theorem. 4

**Que-3** **Attempt any three.** 12

(a) Using Cayley-Hemilton theorem find the matrix for expression $A^5-A^4+3A^3-5A^2+2A-3I$, Where $A=\begin{bmatrix}1&2&3\\-1&0&3\\3&1&-2\end{bmatrix}$

(b) Find Z-transform of $\sin(3n+5)$

(c) Define Unitary matrix and prove that $\frac{1}{2}\begin{bmatrix}1+i&-1+i\\1+i&1-i\end{bmatrix}$ is a unitary matrix

(d) Find Z-transform of $n^2e^{n\theta}$

## Section-II

**Que-4 Answer the following.** 12

(a) Find the directional derivative of $div\ \bar{F}$ at the point (2,2,1) in the direction of normal to the sphere $x^2 + y^2 + z^2 = 9$ where $\bar{F} = x^2zi + xy^2j + yz^2k$

(b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2).

(c) Prove that $f(r)\bar{r}$ is irrotational.

**OR**

**Que-4 Answer the following.** 12

(a) Show that $\bar{F} = 2xyzi + (x^2z + 2y)j + x^2yk$ is irrotational and find its scalar potential.

(b) Find the unit tangent vector at any point on the curve $\bar{r} = (t^2 + 2)i + (4t - 5)j + (2t^2 - 6t)k$. Also determine the same at the point $t = 2$.

(c) Find the direction from the point (1,1,0) which gives the greatest rate of increase of the function $\emptyset = (x + 3y)^2 + (2y - z)^2$.

**Que-5 Answer the following.** 11

(a) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = \frac{yi - xj}{x^2 + y^2}$ and C being the circle $x^2 + y^2 = 1$ traversed counter clockwise. 3

(b) Evaluate the integral $\oint_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$ along the boundary of C of the region $y^2 = 8x$ and $x = 2$. 4

(c) Use Gauss' divergence theorem for $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ over the surface of the rectangular parallelepiped, $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. 4

**OR**

**Que-5 Answer the following.** 11

(a) Find the work done in moving a particle in the force $\bar{F} = 3x^2i + (2xz - y)j + zk$ along the straight line from (0,0,0) to (2,1,3). 3

(b) Use Green theorem to evaluate $\oint_C [x^2ydx + x^2dy]$ where C is the boundary described counter clockwise of a triangle with vertices(0,0), (1,0), (1,1). 4

(c) Evaluate $\iint_S curl\ \bar{F} \cdot \hat{n}\, ds$ where $\bar{F} = y^2i + yj - xzk$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and $z \ge 0$. 4

**Que-6 Attempt any three.** 12

(a) If $\bar{V} = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$ then find value of $div\bar{V}$.

(b) If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$ then prove that $div(\bar{a} \times \bar{r}) = 0$.

(c) If $\bar{F} = (2x^2 - 3z)i - 2xyj - 4xk$ then evaluate $\iiint \nabla \cdot \bar{F} dV$ where $V$ is bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.

(d) Find the work done by a force $yi + xj$ which displaces from origin to a point (i+j).

End of Paper