Seat No.__________

GANPAT UNIVERSITY

B. TECH. SEMESTER: III OPEN ELECTIVE (ALL)

CBCS REGULAR EXAMINATION NOVEMBER- 2014

SUB: (2OS301) VECTOR CALCULUS & Z TRANSFORM

Time: 3 Hours **Total Marks:** 70

Instruction: 1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

Section-I

Que-1 Answer the following. **12**

(a) Using Caley-Hamilton theorem, find A^3, A^{-2} for matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(b) Let $A = \begin{bmatrix} 0 & -2 & 2 \\ 1 & 2 & 0 \\ 3 & 2 & 0 \end{bmatrix}$. If possible obtain the matrix P such that $P^{-1}AP$ is diagonal matrix.

(c) Find eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

OR

Que-1 Answer the following. **12**

(a) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ then find $A^7 - 2A^6 + 4A^5 - 6A^4 + 2A^3 + A^2 - 5A + I$.

(b) Is the matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ diagonalizable? Justify?

(c) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

Que-2 Answer the following.

(a) Define Z-transform and prove that $Z\{n^p\} = -z\frac{d}{dz}Z\{n^{p-1}\}$, where p being positive integer. **4**

(b) Evaluate Z-transform of $\{c^k \cos k \propto\}, k \geq 0$. **4**

(c) Prove that $Z\left\{\cosh\left(\frac{n\pi}{2} + \theta\right)\right\} = \frac{z^2 \cosh\theta - z\cosh\left(\frac{\pi}{2} - \theta\right)}{z^2 - 2z\cosh\frac{\pi}{2} + 1}$ **3**

OR

Que-2 Answer the following.

(a) Define Z-transform and derive Z-transform of a^n. 4

(b) Evaluate Z-transform of $(n^2 2^{-n})$. 4

(c) Find $Z\{sin\ n\theta\}$. 3

12

Que-3 Attempt any two.

(a) Define Hermition and Skew-Hermition matrix. Show that matrix

$A = \begin{bmatrix} -2i & 1+2i \\ -1+2i & i \end{bmatrix}$ is Skew-Hermition matrix and find its eigen values.

(b) Find $mult_A$ and $mult_G$ for each eigen values of matrix $A = \begin{bmatrix} 3 & -2 & 2 \\ -2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$.

(c) Define inverse Z-transform and evaluate inverse Z-transform of $\left\{\frac{z^3-20z}{(z-2)^3(z-4)}\right\}$.

Section-II

12

Que-4 Answer the following.

(a) Find directional derivatives of $\emptyset = 3e^{2x-y+z}$ at $A(1,1,-1)$ in the direction of $\overrightarrow{AB}$ where B is the point $(-3,5,6)$.

(b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2).

(c) Find the unit tangent vector at any point on the curve $\bar{r} = (t^2+2)i + (4t-5)j + (2t^2-6t)k$. Also determine the same at the point $t = 2$.

OR

12

Que-4 Answer the following.

(a) Show that $\bar{F} = 2xyzi + (x^2z + 2y)j + x^2yk$ is irrotational and find its scalar potential.

(b) Find the direction from the point (1,1,0) which gives the greatest rate of increase of the function $\emptyset = (x+3y)^2 + (2y-z)^2$.

(c) Find the divergence and curl of the vector function $\bar{F} = xyzi + 3x^2yj + (xz^2 - y^2z)k$ at the point $(2,-1,1)$.

Que-5 Answer the following. 11

(a) If $\bar{F} = 2i + 2zj + yk$, evaluate $\iiint_V \bar{F}\, dV$ where V is the region bounded by the surface $x^2 + y^2 = 9$ and the planes $z = 0, z = 4$. 4

(b) Use Gauss' divergence theorem for $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ over the surface of the rectangular parallelepiped, $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. 4

(c) Use Green's theorem evaluate $\oint_C [(xy - x^2)\, dx + x^2 y\, dy]$ along the closed curve C formed by $y = 0, x = 1$ and $y = x$. 3

OR

Que-5 Answer the following. 11

(a) If V is the region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + y + z = 2$, and $\bar{F} = 2zi + yk$ then evaluate $\iiint_V \bar{F}\, dV$. 4

(b) If $\bar{F} = (2y + 3)i + xzj + (yz - x)k$, evaluate $\int_C \bar{F} \cdot d\bar{r}$ along the straight line joining (0,0,0) to (2,1,1). 4

(c) Find the work done by a force $yi + xj$ which displaces from origin to a point $(i + j)$. 3

Que-6 Attempt any two. 12

(a) Prove that $\nabla \cdot \left\{ r\, \nabla \left(\frac{1}{r^3} \right) \right\} = 3r^{-4}$

(b) Use Gauss' divergence theorem to evaluate $\iint_S \bar{F} \cdot \hat{n}\, d\bar{s}$ where $\bar{F} = yi + xj + z^2 k$ and S is the surface bounding the region $x^2 + y^2 = a^2$, $z = 0$ and $z = h$.

(c) Verify Green's theorem for the function $\bar{F} = (x^2 + y^2)i - 2xyj$ and C is the rectangle in the xy −plane bounded by $y = 0, y = b, x = 0$ and $x = a$.

END OF PAPER