morning
Date: 27/05/2015.

Seat No. ______________

GANPAT UNIVERSITY

B.Tech Semester – IV (ALL) Regular Examination April - June 2015

Subject with Code: (2OS401) VectorCalculus & Z – Transform

Time: 3 hrs. **Marks: 70**

1. All questions are compulsory.
2. Write answer of each section in separate answer books.
3. Figures to the right indicate marks of questions.

Section – I

Que – 1

(A) Find eigen values & eigen vector for matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ (5)

(B) Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ (5)

(C) Define Skew – Hermitian matrix with an example. (2)

OR

(A) Is the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is diagonalisable? (6)

(B) Obtain $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ (6)

Que – 2

(A) Prove: $Z(n^p) = (-z)\frac{d}{dz}\{Z(n^{p-1})\}$ and obtain expressions for $Z(n)$ and $Z(n^2)$. (6)

(B) Derive $Z(\sin n\theta) = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$ (3)

(C) State and prove Damping rules. (2)

OR

Que – 2

(A) State and prove Final value theorem and convolution theorm. (6)

(B) Find $Z\left\{\frac{1}{(z-3)(z-2)}\right\}$ in the following domains (5)

(i) $|z| < 2$ (ii) $2 < |z| < 3$ (iii) $|z| > 3$

Que – 3 Attempt any Three

(A) If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$ then evaluate u_0, u_1, u_2 and u_3. (4)

(B) Prove $Z(n \cdot u_n) = (-z)\frac{d}{dz}\{U(z)\}$ & using it find $Z(n \cdot \sin\theta)$ (4)

(C) If $A = \begin{bmatrix} 3+i & 3+i \\ -3+i & 3-i \end{bmatrix}$ is $\frac{1}{20} \cdot A \cdot A^* = I_2$? (4)

(D) Express $A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as sum of hermitian and skew – hermitian matrix. (4)

Section – II

Que – 4

(A) Define del and gradient of scalar function. If $f(x, y, z) = xyz$ then find ∇f at point $(1, 2, 3)$. (4)

(B) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = xy + yz + zx$ then prove that $(grad\,u) \cdot [(grad\,v) \times (grad\,w)] = 0$ (4)

(C) Define directional derivative and find it of $f = xy^2 + yz^3$ at point $(2, -1, 1)$ in the direction of vector $i + 2j + 3k$. (4)

OR

Que – 4

(A) Find the angle between the normals to the surface $xy = z^2$ at points $(4, 1, 2)$ and $(3, 3, -3)$. (4)

(B) Find $\nabla\emptyset$, where $\emptyset = (x^2 + y^2 + z^2) \cdot e^{\sqrt{x^2+y^2+z^2}}$ (4)

(C) Prove that $div \cdot [grad\, r^n] = n(n+1) \cdot r^{n-2}$ (4)

Que – 5

(A) Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2 dy]$; where C is bounded by $y = x$ and $y = x^2$. (5)

(B) Verify Stokes' theorem for $F = (x^2 + y^2)i - (2xy)j$ taken arround the rectangle bounded by $x = \pm a$, $y = 0$, $y = b$. (6)

OR

Que – 5

(A) State Green's and Stokes' theorems. Prove that Green's theorem is a particular case of Stokes' theorem. (5)

(B) State Divergence theorem and evalute $\iint \vec{F} \cdot \overline{ds}$ where $\vec{F} = (xy^2)i + (yz^2)j + (zx^2)k$ & S is surface of $x^2 + y^2 + z^2 = 1$ (6)

Que – 6 Attempt any Three

(A) Evaluate $\int_C (2y + 3)i + (xz)j + (yz - x)k \cdot dr$ along line joining origin and point (2 , 1 , 1). (4)

(B) Show that $\vec{F} = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is irrotational and find corresponding scalar point function such that $\vec{F} = \nabla\emptyset$. (4)

(C) Prove that $div\left(\frac{\vec{r}}{r^3}\right) = 0$ where $\vec{r} = xi + yj + zk$ (4)

(D) Determine constants a , b , c and d so that vector function $\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k = 0$ is irrotational. (4)

End of Paper