Student Exam No:_____

GANPAT UNIVERSITY

B.TECH. SEM-IV (OPEN ELECTIVE) (IT/CE/EC/MC/ME/BM&I/Civil/EE)

CBCS REGULAR THEORY EXAMINATION MAY 2014

SUBJECT: 2OS401 - VECTOR CALCULUS & Z TRANSFORM

TIME:-3 HOURS **TOTAL MARKS-70**

INSTRUCTIONS:- (1) All questions are compulsory.
(2) Write answer of each section in separate answer books.
(3) Figures to the right indicate marks of questions.

Section-I

Que-1 **Answer the following.** **12**

(a) If $\phi = x^3 + y^3 + z^3 - 3xyz$ then find $\nabla\phi$ at (1,-2, 3).

(b) Find directional derivatives of $\phi = xy + yz + zx$ at the point $(1, 2, 3)$ in the direction of $3i + 2j + k$

(c) Check whether the field $F = (x^2 - y^2 + x)i - (2xy + y^2)j$ is irrational or not.

OR

Que-1 **Answer the following.** **12**

(a) Find the angle between the surfaces $xy^2z = 3x + z^2$ *and* $3x^2 - y^2 + 2z = 1$ at the point (1,2,1).

(b) If $F = grad(x^3 + y^3 + z^3 - 3xyz)$, find (1) div F (2) curl F

(c) Prove that $F = (x + 3y)i + (y - 3z)j + (x - 2z)k$ is Solenoidal.

Que-2 **Answer the following.**

(a) If a force $\vec{F} = (2x^2y)i + (3xy)j$ displaces a particle in the xy – plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$. Find the work done. **3**

(b) If $F = (2x^2 + y^2)i + (3y - 4x)j$, evaluate $\int F\,dr$ around the ΔABC whose vertices are A(0,0), B(2,0) and C(2,1). **4**

(c) Evaluate by the Green's theorem $\int_C [(xy + y^2)dx + (x^2)dy]$, where C is the boundary of the region bounded by $y = x$ and $y = x^2$. **4**

OR

Que-2 **Answer the following.**

(a) Find the work done in moving a particle in a force field $\vec{F} = (3x^2)i + (2xz - y)j + 2k$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$. **3**

(b) Prove that $\vec{F} = yzi + (xz + 1)j + xyk$ is conservative and find scalar potential. **4**

(c) Evaluate $\iint_S A \cdot \hat{n}\,ds$, Where $A = 18zi - 12j + 3yk$ and S is the part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. **4**

Que-3 **Attempt any three.** **12**

(a) If $\vec{r} = xi + yj + zk$ then prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

(b) Find a, b, c when $V = (x + y + az)i + (bx + 3y - z)j + (3x + cy + x)k$ is irrotational.

(c) If $F = (xy)i - (z)j + (x^2)k$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$ evaluate line integral $\int_c F \cdot dr$

(d) Using Stokes' theorem evaluate $\int_C [(2x - y)dx - yz^2dy - y^2zdz]$, where C is the boundary of the circle $x^2 + y^2 = 1$, corresponding to the surface of sphere of unit radius.

Section-II

Que-4 **Answer the following.** **12**

(a) Verify Cayley-Hamilaton theorem and find inverse for the matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

(b) Define with example (1) Complex Matrix, (2) Conjugate of Matrix

(c) Check whether the matrix $\frac{1}{20}\begin{bmatrix} 3+i & 3+i \\ -3+i & 3-i \end{bmatrix}$ is a unitary matrix or not.

OR

Que-4 **Answer the following.** **12**

(a) Using Cayley-Hamilton theorem find inverse for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

(b) Check whether the matrix $A = \begin{bmatrix} 7 & 3+i & 5-4i \\ 3-i & -8 & 2i \\ 5+4i & -2i & 0 \end{bmatrix}$ is Hermitian or not.

(c) If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, find A^*A.

Que-5 **Answer the following.**

(a) Define Z-Transform and Prove that $Z\left(n^p\right) = -z\frac{d}{dz}Z(n^{p-1})$, p being a positive integer. **3**

(b) Find Z-Transform of $e^t \sin 4t$ **4**

(c) Find inverse Z-transform of $n^2 e^{an}$. **4**

OR

Que-5 **Answer the following.**

(a) Define Z-transform and show that $Z\left(a^n\right) = \frac{z}{z-a}$ **3**

(b) Find Z-transform of $(n+1)^2$ **4**

(c) Find inverse Z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$ using partial fraction method. **4**

Que-6 **Attempt any three** **12**

(a) Using Cayley-Hemilton Theorem find the matrix for expression

$A^6 - 6A^5 + 9A^4 - 2A^3 - 23A - 9I$, Where $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(b) Find Z-transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$

(c) Use Convolution theorem to evaluate $Z^{-1}\left(\frac{z^2}{(z-3)(z-2)}\right)$

(d) Diagonalise the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

END OF PAPER